

Spring 2005 #2

$H = T + V$ $H_s = T + V_s$ H_s, V_s belong to a one dim. attractive square well which always has a bound state.

$$\Rightarrow \int \psi_s^* (T + V_s) \psi_s dx = E_s \quad (1)$$

E_0 is ground state for V_s . Let's use ψ_s^0 as a trial wave function.

$$(2) \int \psi_s^0 (T + V) \psi_s dx \geq E_0 \quad [\text{main point of variation method}]$$

Subtract (1) from (2)

$$\int \psi_s^0 (V - V_s) \psi_s dx \geq E_0 - E_s$$

Since V is negative for all x and V_s always has a bound state no matter what size it is.

$\Rightarrow (V - V_s)$ negative for all x

\Rightarrow negative amount $\geq E_0 - E_s$ or

$$E_0 \leq \text{negative amount} + E_s \quad \text{And } E_s \text{ is negative}$$

$\Rightarrow E_0$ is always negative and therefore bound.