Spring 2005 # 4

$H = g\mu_0 \frac{S}{\hbar} \cdot B \qquad B = B_0 \hat{z} \qquad \hbar = 1 \qquad \sigma = 2S$

a) $H = g\mu_0 B_0 S_z = \frac{g\mu_0 B_0}{2} \sigma_z$

$\lambda = \pm \frac{g\mu_0 B_0}{2} \quad |+\rangle = \begin{pmatrix}1\\0\end{pmatrix} \quad |-\rangle = \begin{pmatrix}0\\1\end{pmatrix}$

$\Delta E = g\mu_0 B_0$

b) $t=0 \quad |\psi(0)\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} = \frac{1}{\sqrt{2}}\left[|+\rangle + |-\rangle\right]$

$\psi(t) = e^{-iHt}|\psi(0)\rangle$

$= \frac{e^{-iHt}}{\sqrt{2}}|+\rangle + \frac{e^{-iHt}}{\sqrt{2}}|-\rangle = \begin{pmatrix} e^{-i g\frac{\mu_0}{2}B_0 t} \\ e^{i g\frac{\mu_0 B_0 t}{2}} \end{pmatrix}\frac{1}{\sqrt{2}} \qquad \omega = \frac{g\mu_0 B_0}{2}$

$\psi(t) = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\omega t} \\ e^{i\omega t}\end{pmatrix}$

c) $\langle S_x \rangle = \langle \frac{\sigma_x}{2} \rangle = \frac{1}{\sqrt{2}}\left(e^{+i\omega t} \; e^{-i\omega t}\right)\frac{1}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}e^{-i\omega t}\\ e^{i\omega t}\end{pmatrix}$

$= \frac{1}{4}\left(e^{+i\omega t} \; e^{-i\omega t}\right)\begin{pmatrix}e^{i\omega t}\\ e^{-i\omega t}\end{pmatrix} = \frac{1}{4}\left(e^{i2\omega t} + e^{-i2\omega t}\right) = \frac{1}{2}\cos(2\omega t)$

$$\langle S_y \rangle = \frac{i}{4} \left(e^{i\omega t} \;\; e^{-i\omega t} \right) \underbrace{\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} e^{-i\omega t} \\ e^{i\omega t} \end{pmatrix}}$$

$$= \frac{i}{4} \left(e^{i\omega t} \;\; e^{-i\omega t} \right) \begin{pmatrix} -e^{i\omega t} \\ e^{-i\omega t} \end{pmatrix} = \frac{i}{4} \left(-e^{i2\omega t} + e^{-i2\omega t} \right)$$

$$= \frac{1}{4i} \left(e^{i2\omega t} - e^{-i2\omega t} \right) = \frac{1}{2} \sin(2\omega t)$$

$$\langle S_z \rangle \underset{\substack{\uparrow \\ \text{should}}}{=} 0 \quad \frac{1}{4} \left(e^{i\omega t} \;\; e^{-i\omega t} \right) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} e^{-i\omega t} \\ e^{i\omega t} \end{pmatrix}$$

$$= \frac{1}{4} \left(e^{i\omega t} \;\; e^{-i\omega t} \right) \begin{pmatrix} e^{-i\omega t} \\ -e^{i\omega t} \end{pmatrix} = \frac{1}{4} \left(1 - 1 \right) = 0 \checkmark$$