Spring 2005 #6

A	B
V/2 N/2	V/2 N/2

a) $Z = \frac{1}{N!}\left(\frac{V}{(\sqrt{2\pi\hbar^2/mk_bT})^3}\right)^N = \frac{1}{N!}\left(\frac{V}{\lambda_{th}^3}\right)^N$

Before

$Z_A = \frac{1}{N_A!}\left(\frac{V_A}{\lambda_{th}^3}\right)^{N_A} \quad Z_B = \frac{1}{N_B!}\left(\frac{V_B}{\lambda_{th}^3}\right)^{N_B}$

After

$Z_{AB} = \frac{1}{N_A!}\left(\frac{V}{\lambda_{th}^3}\right)^{N_A}\frac{1}{N_B!}\left(\frac{V_B}{\lambda_{th}^3}\right)^{N_B}$

$S = k(\ln Z + \beta \bar{E})$

Internal energy

$\frac{3}{2}NkT$

If $Z = \frac{Z_1^N}{N_1}$

$\ln Z = N\ln Z_1 - \ln N!$

$= N\ln Z_1 - N\ln N + N$

$= kN[\ln V + \frac{3}{2}\ln T + \sigma] + k(-N\ln N + N) \qquad \sigma = \frac{3}{2}\ln\left(\frac{2\pi m k}{h^2}\right) + \frac{3}{2}$

$S = kN[\ln\frac{V}{N} + \frac{3}{2}\ln T + \sigma_0] \qquad \sigma_0 = 1 + \sigma$

Before

$S = S_A + S_B = \underbrace{k\frac{N}{2}[\ln\frac{V/2}{N/2} + \frac{3}{2}\ln T + \sigma_0]}_{S_A} + \underbrace{\frac{kN}{2}[\ln\frac{V/2}{N/2} + \frac{3}{2}\ln T + \sigma_0]}_{S_B}$

$= kN[\ln\frac{V}{N} + \frac{3}{2}\ln T + \sigma_0]$

After

$S = S_A + S_B = k\frac{N}{2}[\ln\frac{V}{N/2} + \frac{3}{2}\ln T + \sigma_0] + \frac{kN}{2}[\ln\frac{V}{N/2} + \frac{3}{2}\ln T + \sigma_0]$

$= kN[\ln\frac{V}{N} + \frac{3}{2}\ln T + \sigma_0] + Nk\ln 2$

$$S_{after} - S_{before} = NK \ln 2$$

b) If you remove the wall, no work is done

$\Rightarrow \Delta E = \Delta Q$ But since the energy depends only on the temperature for ideal gases and T is fixed $\Delta E = 0 \Rightarrow \Delta Q = 0$

The process is irreversable though since $\Delta S \neq 0$.

c) $\Delta Q = 0$ still for the same reason.

$\Delta S = 0$ Now so the process is reversable which makes sence with the particles all being identical. Basicly nothing happened.