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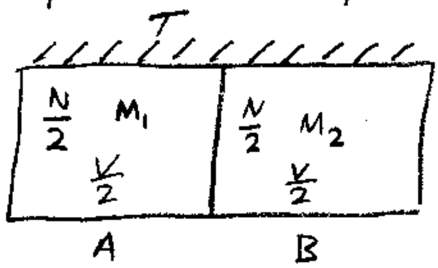
A closed container is divided by a wall into two equal parts like the figure



where M_1 & M_2 are identical types of particles but distinguishable from each other and they both make up ideal gases.

a) the partition function $Z(N) = \frac{1}{N!}\left(\frac{V}{\sqrt{2\pi\hbar^2/MkT}}\right)^N$ is for an ideal gas of N particles of mass M in a volume V. Give the partition function of the gas in the container before and after the wall is removed. What are the entropy and pressure before and after the wall is removed?

Before wall is removed, we simply have

$$Z_A = \frac{1}{(N/2)!}\left(\frac{V/2}{\sqrt{2\pi\hbar^2/M_1kT}}\right)^{N/2}$$

$$Z_B = \frac{1}{(N/2)!}\left(\frac{V/2}{\sqrt{2\pi\hbar^2/M_2kT}}\right)^{N/2}$$

Set $\hbar = 1$.

So, the partition function for the system is

$$Z = Z_A Z_B = \frac{1}{[(N/2)!]^2}\left[\frac{V^2/4}{\frac{2\pi}{kT}\sqrt{\frac{1}{M_1M_2}}}\right]^{N/2}$$

$$\therefore \boxed{Z_{before} = \frac{1}{[(N/2)!]^2}\left[\frac{V(M_1M_2)^{1/4}}{2\sqrt{\beta 2\pi}}\right]^N}$$

→ note partition functions of uncorrelated systems are multiplied by each other

After the wall is removed, the partition function is (now $\frac{V}{2} \to V$)

$$\boxed{Z_{after} = \frac{1}{[(N/2)!]^2}\left[V\sqrt{\frac{(M_1M_2)^{1/2}}{2\pi\beta}}\right]^N}$$

Now, we want to find the entropy and pressure. To do this, let's first find the free energy. So,

$$F = -kT\ln Z \quad \text{and} \quad p = -\left(\frac{\partial F}{\partial V}\right)_{T,N} \quad \& \quad S = -\left(\frac{\partial F}{\partial T}\right)_{V,N}$$

where

$$\ln Z_{before} = -2\ln\left(\frac{N}{2}\right)! + N\ln\frac{V(M_1M_2)^{1/4}}{2\sqrt{\beta 2\pi}} \qquad , N \gg 1$$

$$\cong -N\ln\frac{N}{2} + N + N\ln V + N\ln\frac{(M_1M_2)^{1/4}}{2\sqrt{2\pi\beta}} + \frac{1}{2}N\ln kT$$

and

$$\ln Z_{after} \cong -N\ln\frac{N}{2} + N + N\ln V + \frac{N}{2}\ln\frac{(M_1M_2)^{1/2}}{2\pi\beta} + \frac{N}{2}\ln kT$$

So,

$$P_{before} = kT\left(\frac{\partial \ln Z_{before}}{\partial V}\right)_{T,N} = \boxed{\frac{kTN}{V}}$$

and

$$P_{after} = kT\left(\frac{\partial \ln Z_{after}}{\partial V}\right)_{T,N} = \boxed{\frac{kTN}{V}}$$

} pressure is constant!

Also, we know that $S = -\left(\frac{\partial F}{\partial T}\right)_{V,N}$. But the following will be easier to use

$$S = k(\ln Z + \beta\bar{E}) \qquad \text{(Ref eq 6.6.5)}$$

So,

$$S_i = k\left(\ln Z_i + \beta\bar{E}\right) \quad , \bar{E} = \frac{3}{2}\left(\frac{N}{2}\right)kT$$

Then,

$$S_{before} = S_A + S_B = K\left(\ln Z_{A_{before}} + \frac{3}{2}\frac{N}{2}\right) + K\left(\ln Z_{B_{before}} + \frac{3}{2}\left(\frac{N}{2}\right)\right)$$

$$= K\ln Z_{A\,before} Z_{B\,before} + \frac{3}{2}NK = K\ln Z_{before} + \frac{3}{2}NK$$

$$\boxed{S_{before} = KN\left[-\ln\frac{N}{2} + \frac{5}{2} + N\ln\frac{V(m_1m_2)^{1/4}}{2\sqrt{\beta 2\pi}}\right]}$$

And

$$\boxed{S_{after} = S_{before} + KT\ln 2}$$

Thus,

$$\Delta S = S_{after} - S_{before} = KT\ln 2$$

(b) How much heat is absorbed or released following the removal of the wall? Is it reversible or irreversible process?

Since $\Delta S > 0$, then the process is irreversible. Since no heat is exchanged with the environment $dS \neq \frac{dQ}{T}$, so, we must consider 1st law of thermo (no work is done)

$$dE = dQ$$

Since $dE = 0$, $dQ = 0$.

(c) let $M_1 = M_2$ and answer the same question as part (b).

Since the particles are no identicle, $\Delta S = 0$. The process is reversible, we still have $dQ = 0$.