Spring 1995 # 7

By heating a metal we produce thermal electrons. Recall that a dilute electron gas can be approximated as a monatomic ideal gas with free energy given by

$$F(T, V, N) = NkT\left(\ln\left(\frac{N}{V}\right) - \frac{3}{2}\ln(T) - c\right)$$

where c is a constant. Inside the metal, the free energy of the electrons can be approximated by

$$F(T, V, N) = -N\phi$$

where ϕ is the work function. Derive the expression for the vapor pressure $p(T)$ of the electron gas in thermal equilibrium with the metal at temperature T.

→ in equilibrium, the chemical potentials will be equal.

$$\mu = \left(\frac{\partial F}{\partial N}\right)_{T,V}$$

Thus,

$$\mu_{metal} = -\phi \qquad \mu_{gas} = kT\left(\ln\left(\frac{N}{V}\right) - \frac{3}{2}\ln(T) - c\right) + \cancel{N}kT\left(\frac{1}{\cancel{N}}\right)$$

$$-\phi = kT\left(\ln\left(\frac{N}{V}\right) - \frac{3}{2}\ln T - c\right) + kT$$

if electron gas behaves as ideal gas, $pV = NkT \Rightarrow \frac{N}{V} = \frac{p}{kT}$

$$-\frac{\phi}{kT} = \ln\left(\frac{N}{V}\right) - \frac{3}{2}\ln(T) - c + 1$$

$$\ln\left(\frac{p}{kT}\right) = \frac{-\phi}{kT} + \frac{3}{2}\ln(T) + c - 1$$

$$\frac{p}{kT} = \exp\left(-\frac{\phi}{kT} + \frac{3}{2}\ln T + c - 1\right) = P_o T^{3/2} e^{-\phi/kT}$$

where $P_o = e^{(c-1)}$

$$\boxed{P(T) = P_o k T^{5/2} e^{-\phi/kT}}$$

for large T, $e^{-\phi/kT} \approx 1$ so $\boxed{P(T \gg 1) \approx P_o k T^{5/2}}$

for small T, $T^{5/2} \approx 0$ so $\boxed{P(T \ll 1) \approx 0}$