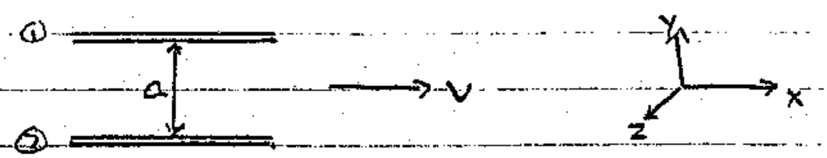
Fall 1996 #3

Two thin, parallel, infinitely long, non-conducting rods, a distance a apart, with identical constant charge density λ per unit length in the rest frame, move with velocity v, not necessarily small compared to the speed of light. Calculate the force per unit length between them in a frame of reference that is at rest, a[nd] in a frame of reference moving with the rods, and compare.



→ in the frame where the rods are at rest, there is no magnetic field; the electric field can be calculated using Gauss's Law

$$\oint \vec{E}\cdot d\vec{a} = 4\pi Q_{enc}$$

$$E\cdot 2\pi r L = 4\pi\lambda L \qquad \Rightarrow \quad \vec{E} = \frac{2\lambda}{r}\hat{r}$$

(where $r=\sqrt{y^2+z^2}$, $\hat{r}=\hat{y}+\hat{z}$)

Thus the force experienced by each rod has magnitude

$$\boxed{|F| = \frac{2\lambda^2}{a} \qquad \text{the force is repulsive}}$$

→ in the frame where the rods move, we must perform a transformati[on] to find the new $\vec{E},\vec{B}$ fields. The tranformation takes the form

$$\vec{E}_\perp' = \gamma\left(E_\perp + \frac{\vec{v}}{c}\times\vec{B}\right) \qquad \vec{B}_\perp' = \gamma\left(\vec{B}_\perp - \frac{\vec{v}}{c}\times\vec{E}\right)$$

$$\vec{E}_\parallel' = \vec{E}_\parallel \qquad \vec{B}_\parallel' = \vec{B}_\parallel$$

(Greiner eq 22.33)

where $\parallel$, $\perp$ refer to motion of system

for motion in the x direction ($\vec{v} = v\hat{x}$) we get the tranformations

$$E_x' = E_x = 0 \qquad E_y' = \gamma E_y \qquad E_z' = \gamma E_z$$

$$B_x' = B_x = 0 \qquad B_y' = +\frac{v}{c}E_z \qquad B_z' = -\frac{v}{c}E_y$$

so since the rest frame E field is $E = \frac{2\lambda}{r}(\hat{y} + \hat{z})$
the new fields are (for a single rod)

$$E_x' = 0 \qquad E_y' = \frac{2\lambda\gamma}{r}\hat{y} \qquad E_z' = \frac{2\lambda\gamma}{r}\hat{z}$$

$$B_x' = 0 \qquad B_y' = \frac{2v\lambda\gamma}{cr}\hat{y} \qquad B_z' = -\frac{2v\lambda\gamma}{cr}\hat{z}$$

Let's find the force on rod 2
the new electric force is still repulsive and given by:

$$\boxed{F_{elect} = \frac{-2\lambda^2\gamma^2}{a}\hat{y}}$$ since the charge density of a rod becomes $\lambda\gamma$

Next, find the magnetic force, given by $F_{mag} = \frac{1}{c}(I \times B = \frac{1}{c}(\lambda\bar{v}) \times \vec{B})$

the charge density again becomes $\lambda\gamma$
the force will act in the y direction, so we calculate

$$F_{mag} = \frac{1}{c}(\lambda\gamma v\hat{x}) \times \left(\frac{2v\lambda\gamma}{ca}(-\hat{z})\right)$$

$$\boxed{F_{mag} = \frac{2\lambda^2\gamma^2}{a}\frac{v^2}{c^2}\hat{y}}$$

What happened to B_y'? Did it cancel E_z'?

adding the forces yields

$$\vec{F}_{tot} = -\frac{2\lambda^2\gamma^2}{a}\underbrace{\left(1 - \frac{v^2}{c^2}\right)}_{1/\gamma^2}\hat{y}$$

$$\boxed{F_{tot} = \frac{-2\lambda^2}{a}\hat{y}}$$ the force is identical to that found in