# Spring 1996 #14



Consider the 3-dimensional harmonic oscillator described by the Hamiltonian:

$$H_0 = \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2) + \frac{1}{2}m\omega^2(x^2+y^2+z^2)$$

A small pertubation $V = \lambda xyz$ is introduced. Compute to lowest nonvanishing order the shift in energy of the ground state of the unperturbed Hamiltonian.

→ The perturbation can be rewritten as:

$$V = \lambda xyz = \frac{\lambda}{(2m\omega)^{3/2}}(a_x + a_x^+)(a_y + a_y^+)(a_z + a_z^+)$$

where $a$, $a^+$ are lowering, raising operators, i.e. $a|n\rangle = \sqrt{n}|n-1$
$a^+|n\rangle = \sqrt{n+1}\,|n+$

the ground state of the 3-D oscillator can be written $|n_x=0, n_y=0, n_z$
and the energy levels are: $E = \hbar\omega(n_x + n_y + n_z + 3/2)$, $n_i = 0,1,2\cdots$

$$V = \frac{\lambda}{(2m\omega)^{3/2}}(a_x a_y + a_x a_y^+ + a_x^+ a_y + a_x^+ a_y^+)(a_z + a_z^+)$$

$$V = \frac{\lambda}{(2m\omega)^{3/2}}(a_x a_y a_z + a_x a_y^+ a_z + a_x^+ a_y a_z + a_x^+ a_y^+ a_z + a_x a_y a_z^+ + a_x a_y^+ a_z^+ + a_x^+ a_y a_z^+ + a_x^+ a_y^+$$

the first order correction is given by

$$E_0^1 = \frac{\lambda}{(2m\omega)^{3/2}} \langle n_x=0, n_y=0, n_z=0 | V | n_x=0, n_y=0, n_z=0\rangle$$

$$E_0^1 = 0 \quad \text{since } \langle n|a^+|n\rangle = 0, \ \langle n|a|n\rangle = 0$$

the 2nd order correction is given by

$$E_n^2 = \sum_{m\neq n} \frac{|\langle \psi_m^0 | H' | \psi_n^0\rangle|^2}{E_n^0 - E_m^0} \qquad \text{(Griffiths eq 6.14)}$$

the only $\psi_m^0$ that will give a nonzero inner product is $|n_x=1, n_y=1, n_z=1\rangle$ when acted on by $a_x a_y a_z$

so $|\langle \psi_m^0 | a_x a_y a_z | \psi_n^0\rangle|^2 = \frac{\lambda^2}{(2m\omega)^3}$

$$E_n^\circ = \hbar\omega(0+0+0+3/2) = \frac{3\hbar\omega}{2}$$

$$E_m^\circ = \hbar\omega(1+1+1+3/2) = \frac{9}{2}\hbar\omega$$

$$E_n^\circ - E_m^\circ = -3\hbar\omega$$

thus $\boxed{E_0^2 = \dfrac{-\lambda^2}{(2m\omega)^3\, 3\hbar\omega}}$