Two long, coaxial cylindrical conducting surfaces of radii a and b are lowered vertically into a dielectric liquid. A potential difference V is applied between the cylindrical surfaces.

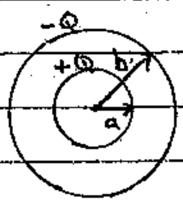
a) Will the dielectric liquid rise or fall between the electrodes when the potential difference is applied?

(refer to Griffith's pgs 188-189)

When the potential difference is applied, the potential will induce charge in the dielectric, which will then be attracted to the plates. Thus;

$$\boxed{\text{The dielectric will rise}}$$

To calculate the force, first find the capacitance of the cylinders:



Applying Gauss's Law for $a<r<b$, one finds (neglecting ends)

$$E(2\pi r L) = 4\pi Q \Rightarrow E = \frac{2Q}{rL}\hat{r} \quad \text{for } a<r<b$$

The potential difference is

$$|V| = +\int_a^b \frac{2Q}{rL}\,dr = \frac{2Q}{L}\ln(b/a)$$

$$\boxed{C = \frac{Q}{V} = \frac{L}{2\ln(b/a)}}$$

now for a capacitor partly filled with dielectric, the capacitance is

$$C = \frac{1}{2\ln(b/a)}(L + \chi_e h)$$ where h = height of dielectric (Griffiths eq 4.47)

If we were to change the height by dh, we would need to do work $dW = F_{us}\,dh$

where F_{us} is just equal to the force of the capacitor pulling on the dielectric. If the potential is held constant by a battery, the charge on the plates will change as well and the battery will do work $dW = VdQ$. Thus the total work done is

$$dW = F_{us}\,dh + VdQ$$

or in terms of the force applied to the dielectric by the capacitor

$$dW = -F dh + VdQ \quad \Longrightarrow \quad F = -\frac{dW}{dh} + V\frac{dQ}{dh}$$

now $W = \frac{1}{2} C V^2 = \frac{1}{2} V^2 \left(\frac{1}{2\ln(b/a)}\right)(L + \chi_e h)$

$$C = \frac{Q}{V} \Rightarrow Q = CV = V\left(\frac{1}{2\ln(b/a)}\right)(L + \chi_e h)$$

so the force is

$$F = -\frac{1}{2} V^2 \left(\frac{1}{2\ln(b/a)}\right)\chi_e + V^2\left(\frac{1}{2\ln(b/a)}\right)\chi_e$$

$$\boxed{F = V^2\left(\frac{1}{2\ln(b/a)}\right)\chi_e}$$

b) Relate the electric susceptibility to the parameters of the prob

→ In equilibrium, the upward force is balanced by the gravitatio force.

→ The gravitational potential energy of a disk of dielectric

$$\Delta W = \Delta M g h = \pi(b^2 - a^2)\rho g h \,\Delta h$$

giving a downward force of

$$F_{down} = \frac{\Delta W}{\Delta h} = \pi(b^2 - a^2)\rho g h$$

equating the forces yields

$$V^2\left(\frac{1}{2\ln(b/a)}\right)\chi_e = \pi(b^2 - a^2)\rho g h$$

$$\boxed{\chi_e = \frac{2\pi(b^2 - a^2)\rho g h \ln(b/a)}{V^2}}$$