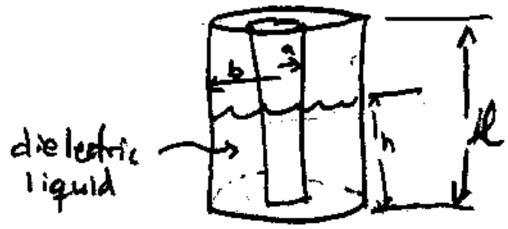
Two long co-axial, cylindrical conducting surfaces of radii a and b (>a) are lowered vertically into a dielectric liquid. A potential difference V is applied between the cylindrical surfaces.

(a) Will the dielectric liquid rise or fall between the electrodes when the potential difference is applied? Give a detailed explanation of the phenomenon.

→ this is "exactly" like Griffiths problem 4.28.

$\boxed{\text{The dielectric liquid will rise!}}$

why?



First find the capacitance as a function of h since $F = \frac{1}{2}V^2 \frac{dC}{dh}$ (Griffiths eq 4.64) see p 193-196 for nice discussion on Forces on Dielectrics

$$C = \frac{Q}{V}$$

So, now let's find Q and V.

let λ be the charge per unit Length of air between cylinders
let λ' be the charge per unit length of liquid

So,

$$Q = \lambda(\ell - h) + \lambda' h \qquad (1)$$

and

$$V_{air} = \frac{2\lambda}{4\pi\epsilon_0}\ln\left(\frac{b}{a}\right) \quad \& \quad V_{liq} = \frac{2\lambda'}{4\pi\epsilon}\ln\left(\frac{b}{a}\right)$$

Since $V_{air} = V_{liq}$, we have

$$\frac{2\lambda}{4\pi\epsilon_0}\ln\left(\frac{b}{a}\right) = \frac{2\lambda'}{4\pi\epsilon}\ln\left(\frac{b}{a}\right) \Rightarrow \boxed{\lambda' = \epsilon_r \lambda}, \quad \epsilon_r = \frac{\epsilon}{\epsilon_0} \qquad (2)$$

Substituting eq (2) into eq (1), we get

$$Q = \lambda(\ell - h + \epsilon_r h) = \lambda(\ell + \chi_e h), \quad \chi_e = \epsilon_r - 1$$

So,

$$C = \frac{Q}{V} = \frac{\lambda(\chi_e h + \ell)}{2\lambda \ln(b/a)} 4\pi\epsilon_0 = 2\pi\epsilon_0 \frac{(\chi_e h + \ell)}{\ln(b/a)}$$

So, the force on the liquid is

$$F = \frac{1}{2} V^2 \frac{dC}{dh} = \frac{1}{2}\left[2\lambda \ln(b/a)\right]^2 \frac{2\pi\epsilon_0}{\ln(b/a)} \chi_e$$

(3) $$\Rightarrow \boxed{F = \frac{\pi\epsilon_0 V^2 \chi_e}{\ln(b/a)}}$$ ← since > 0, Force is upward!

b) Relate the electric susceptibility χ_e of the liquid to the change in height, h, of the liquid, its density, ρ, the acceleration due to gravity g, and the other parameters in the problem.

The gravitational force is

$$F = mg = \rho V g = \rho \pi (b^2 - a^2) h g \quad (4)$$

set eq (3) equal to eq (4) and solve for χ_e.

$$\chi_e = \frac{\rho\pi(b^2-a^2)hg}{\pi\epsilon_0 V^2} \ln(b/a)$$

$$\therefore \boxed{\chi_e = \frac{\rho h g}{\epsilon_0 V^2}(b^2-a^2)\ln(b/a)}$$