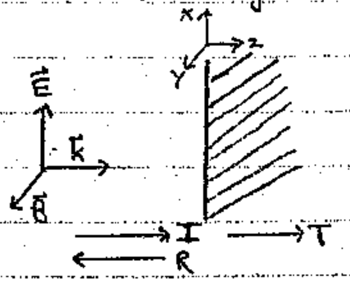
Fall 1997 #7

Linearly polarized light of form $E_x(z,t) = E_0 e^{i(kz-\omega t)}$ is incident normally onto a material which has an index of refraction n_r for right-hand circularly polarized light and n_ℓ for left-hand circularly polarized light. Calculate the intensity and polarization of the reflected light:



For the incoming wave, one can write:

$$\vec{E}_I = E_I e^{i(kz-\omega t)} \hat{x}$$
$$\vec{B}_I = E_I e^{i(kz-\omega t)} \hat{y} \qquad (\text{since } \vec{B} = \hat{k} \times \vec{E}) \quad (\text{Griffiths pg 33}$$

Next, we want to write the incoming wave in terms of circularly polarized waves. Recall that the polarization vectors of circularly polarized waves can be written

$$\hat{\epsilon}_\pm = \frac{1}{\sqrt{2}}(\hat{\epsilon}_1 \pm i\hat{\epsilon}_2) \qquad (\text{Jackson eq } 7.22)$$

In the given case, the polarization vector lies in the x-y plane, so we can write

$$\hat{\epsilon}_+ = \frac{1}{\sqrt{2}}(\hat{x} + i\hat{y}) \qquad (\text{left circular polarization}$$
$$\hat{\epsilon}_- = \frac{1}{\sqrt{2}}(\hat{x} - i\hat{y}) \qquad (\text{right circular polarization}$$

now solve for $\hat{x}$:

$$\hat{\epsilon}_+ + \hat{\epsilon}_- = \frac{1}{\sqrt{2}}(2\hat{x}) \implies \hat{x} = \frac{1}{\sqrt{2}}(\hat{\epsilon}_+ + \hat{\epsilon}_-)$$

So the incident E field is

$$\vec{E}_I = \frac{E_I}{\sqrt{2}} e^{i(kz-\omega t)} (\hat{\epsilon}_+ + \hat{\epsilon}_-)$$

Now apply boundary conditions. Since no free charges or current are present these conditions are $\vec{D}_\perp, \vec{E}_\parallel, \vec{B}_\perp, \vec{H}_\parallel$ all continuous. The conditions for $\vec{D}, \vec{B}$ are already satisfied, leaving $\vec{E}$ and $\vec{H}$

For $E_\parallel$ continuous:

$$E_I + E_R = E_T$$

For $H_\parallel$ continuous:

$$E_I - E_R = n E_T$$

eliminating E_T yields

$$E_I - E_R = n E_I + n E_R$$

$$E_R(n+1) = E_I(1-n) \Rightarrow \boxed{E_R = E_I \frac{(1-n)}{(1+n)}}$$

the equation for E_R applied to both left and right circular polarizatio
The reflected $\vec{E}$ field is thus:

$$\vec{E}_R = \frac{E_I}{\sqrt{2}} e^{i(-kz-\omega t)} \left[\left(\frac{1-n_\ell}{1+n_\ell} \right) \hat{\epsilon}_+ + \left(\frac{1-n_r}{1+n_r} \right) \hat{\epsilon}_- \right]$$

or, in terms of the linear polarization vectors:

$$\boxed{\vec{E}_R = \frac{E_I}{2} e^{i(-kz-\omega t)} \left[\left[\left(\frac{1-n_\ell}{1+n_\ell} \right) + \left(\frac{1-n_r}{1+n_r} \right) \right] \hat{x} + i \left[\left(\frac{1-n_\ell}{1+n_\ell} \right) - \left(\frac{1-n_r}{1+n_r} \right) \right] \hat{y} \right]}$$

So the polarization of the reflected wave is mixed.

$$\text{Intensity} = |E_r|^2$$

$$\boxed{\text{Intensity} = \frac{E_I^2}{2} \left[\left(\frac{1-n_\ell}{1+n_\ell} \right)^2 + \left(\frac{1-n_r}{1+n_r} \right)^2 \right]}$$