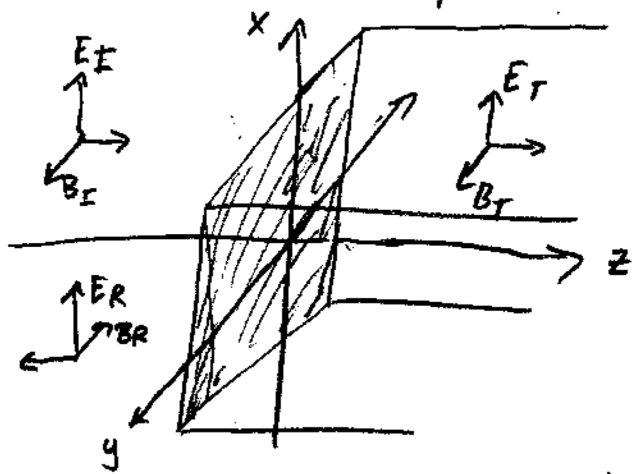
Fall 1997 # 7 (p 1 of 4)

Linearly polarized light of the form $E_x(z,t) = E_0 e^{i(kz-\omega t)}$ is incident normally onto a material which has an index of refraction n_r for right-hand circularly polarized light and n_ℓ for left-hand circularly polarized light. Calculate the intensity and polarization of the reflected light.



general Boundary conditions

$$\epsilon_1 E_1^\perp = \epsilon_2 E_2^\perp$$

$$E_1^\parallel = E_2^\parallel$$

$$B_1^\perp = B_2^\perp$$

$$\frac{1}{\mu_1} B_1^\parallel = \frac{1}{\mu_2} B_2^\parallel$$

The solution starts on p 3 ... the first part is just discussion!

we are given the linearly polarized light is of the form

$$E_x(z,t) = E_I e^{i(kz-\omega t)}$$

So, we know

$$B_y(z,t) = E_I e^{i(kz-\omega t)}$$

Then, the reflected waves are

$$E_x = E_R e^{i(kz-\omega t)} \quad \leftarrow E^\parallel \text{ continuous}$$

$$B_y = -E_R e^{i(kz-\omega t)} \quad \leftarrow \text{if } E^\parallel \text{ same then sign must change b/c } \vec{S} \propto \vec{E} \times \vec{B}$$

and, the transmitted waves are

$$E_x = E_T e^{i(kz-\omega t)}$$

$$B_y = E_T e^{i(k'z-\omega t)}$$

These are all valid for linearly polarized (see Griffiths section 9.3.2). Now, for circular polarized (out of phase by 90°) consider the following (see Marion and Heald 2nd edition section 5.3 and Lim Yung-Kuo #4018)

circularly polarized waves are ones with the same amplitude but phase is off by $\frac{\pi}{2}$.
That is a right-circular polarized light is

$$\vec{E}_R(z,t) = \left[\hat{x} + e^{-i\frac{\pi}{2}}\hat{y}\right] e^{i(K_r' z - \omega t)} \quad , K_r' = \frac{\omega}{c} n_r$$

For left-circular polarization: $e^{-i\pi/2} \rightarrow e^{i\pi/2}$ and $K_r' \rightarrow \frac{\omega}{c} n_L \equiv K_L'$

Now, linearly polarized light can be written in terms of right and left circularly polarized waves like so

$$\vec{E}(z,t) = \tfrac{1}{2}\vec{E}_R(z,t) + \tfrac{1}{2}\vec{E}_L(z,t)$$

$$\Rightarrow 2\vec{E} = \left[\hat{x} + e^{-i\frac{\pi}{2}}\hat{y}\right] E_0 e^{i(K_r' z - \omega t)} + \left[\hat{x} + e^{i\frac{\pi}{2}}\hat{y}\right] E_0 e^{i(K_L' z - \omega t)}$$

where $e^{\pm i\pi/2} = \pm i$ (of course ☺)

So, if $n_r = n_L$ and substituting in for $e^{\mp i\pi/2}$, we easily see that we get back the linearly polarized wave $\vec{E} = \hat{x} E_0 e^{i(K' z - \omega t)}$

Now, let's use this to rewrite our E & B fields in terms of circularly polarization: The only waves that "change" are for the transmitted & reflected ones since $n_L \neq n_R$ in the material, so, we have

$$2\vec{E} = \left[\hat{x} + e^{-i\pi/2}\hat{y}\right] E_T e^{i(K_r' z - \omega t)} + \left[\hat{x} + e^{i\frac{\pi}{2}}\hat{y}\right] E_T e^{i(K_L' z - \omega t)}$$

$$2\vec{B} = \left[e^{i\pi/2}\hat{x} + \hat{y}\right] E_T e^{i(K_r z - \omega t)} + \left[e^{i\pi/2}\hat{x} + \hat{y}\right] E_T e^{i(K_L z - \omega t)}$$

Now, let's use B.C.s to start answering the question. This ends the discussion part. Now, let's begin what you really need to answer.

(See Lim Yung-Kuo #4019)

From B.C.'s, we know that at z=0

$$H_1^{\parallel} - H_2^{\parallel} = \vec{K}_F \times \hat{n} = 0$$

$$E_1^{\parallel} = E_2^{\parallel}$$

So,

$$\underbrace{E_I + E_R}_{E_1} = \underbrace{E_T}_{E_2} \quad \& \quad \overbrace{H_I - H_R}^{H_1} = \overbrace{H_T}^{H_2}$$

↑ sign from change in direction

note: $\vec{B} = n \frac{\vec{k} \times \vec{E}}{k}$ (Jackson 2nd ed 7.18) $\Rightarrow B = nE$

Since we are not given any other info, we will assume that $n = 1$ outside medium and $\mu = 1$ everywhere. So, $B = H$, Thus $H = nE$ for in the medium and $H = E$ outside it. Then our B.C.s are

$$E_I + E_R = E_T \quad \text{and} \quad E_I - E_R = nE_T$$

substituting E_T from the 1st equation into the second yields

$$E_I - E_R = n(E_I + E_R) = nE_I + nE_R$$

$$\Rightarrow \quad E_R(n+1) = (1-n)E_I$$

$$\therefore \quad E_R = \frac{1-n}{1+n} E_I$$

This relationship holds for either polarization, so,

Left hand: $E_R = \frac{1-n_L}{1+n_L} E_I$ right hand: $E_R = \frac{1-n_R}{1+n_R} E_I$

Now, we just need E_I in terms of circular polarization (see p 2). So, we have the reflected polarization is

$$\vec{E} = \left\{ \frac{1}{2} E_I \left(\frac{1-n_r}{1+n_r} \right) [\hat{x} + e^{-i\pi/2} \hat{y}] + \frac{1}{2} E_I \left(\frac{1-n_L}{1+n_L} \right) [\hat{x} + e^{i\pi/2} \hat{y}] \right\} e^{i(kz-\omega t)}$$

note: $n_L = n_r$ outside material.

$$\therefore \quad \vec{E} = \frac{1}{2} E_I e^{i(kz-\omega t)} \left[\left(\frac{1-n_r}{1+n_r} + \frac{1-n_L}{1+n_L} \right) \hat{x} + i \left(\frac{1-n_L}{1+n_L} - \frac{1-n_r}{1+n_r} \right) \hat{y} \right]$$

the intensity is given by

$$I = |\vec{E}|^2 = \frac{E_I^2}{4} \left[\left(\frac{1-n_r}{1+n_r} + \frac{1-n_L}{1+n_L} \right)^2 + \left(\frac{1-n_L}{1+n_L} - \frac{1-n_r}{1+n_r} \right)^2 \right]$$

$$= \frac{E_I^2}{4} \left[2 \left(\frac{1-n_r}{1+n_r} \right)^2 + 2 \left(\frac{1-n_L}{1+n_L} \right)^2 \right]$$

$$\therefore \quad I = \frac{E_I^2}{2} \left[\left(\frac{1-n_r}{1+n_r} \right) + \left(\frac{1-n_L}{1+n_L} \right) \right]$$