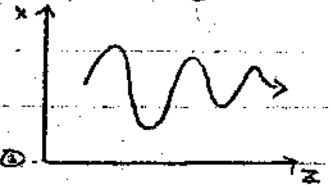
A spacecraft measures that the electric field of an EM wave propagating through the solar wind has the following functional dependence:

$$E(z,t) = E_0 e^{-\alpha z} \cos(kz - \omega t)\,\hat{x}$$



where E_0, α, k, ω are real constants (positive).

a) Deduce the oscillatory current density in this region of the solar wind.

→ first find $\vec{B} = n(\hat{k} \times \vec{E}) = n(\hat{z} \times \hat{x})\, E_0 e^{-\alpha z} \cos(kz - \omega t)$

index of refraction (n)

$$\vec{B} = n E_0 e^{-\alpha z} \cos(kz - \omega t)\,\hat{y}$$

$n = \sqrt{\frac{\epsilon\mu}{\epsilon_0 \mu_0}}$

next, use the Maxwell equation:

$$\nabla \times \vec{H} = \frac{4\pi}{c}\vec{J}_f + \frac{1}{c}\frac{\partial \vec{D}}{\partial t}$$

solve for $\vec{J}$:

$$\vec{J} = \frac{c}{4\pi}\left(\nabla \times \vec{H} - \frac{1}{c}\frac{\partial \vec{D}}{\partial t}\right)$$

$$J = \frac{c}{4\pi}\left\{\left[\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right] \times \sqrt{\frac{\epsilon}{\mu}}\, E_0 e^{-\alpha z} \cos(kz - \omega t)\,\hat{y} - \frac{1}{c}\frac{\partial}{\partial t}\epsilon E_0 e^{-\alpha z} \cos(kz - \omega t)\right.$$

$$J = \frac{c}{4\pi}\left[\sqrt{\frac{\epsilon}{\mu}}\, E_0 (-\alpha) e^{-\alpha z} \cos(kz - \omega t)(\hat{z} \times \hat{y}) + \sqrt{\frac{\epsilon}{\mu}}\, E_0 e^{-\alpha z} k \sin(kz - \omega t)(\hat{z} \times \hat{y})\right.$$

$$\left. + \frac{1}{c}\epsilon E_0 e^{-\alpha z} {-\omega} \sin(kz - \omega t)\,\hat{x}\right]$$

$$J = \frac{c}{4\pi} E_0 e^{-\alpha z}\left[\alpha\sqrt{\frac{\epsilon}{\mu}} \cos(kz - \omega t)\,\hat{x} + \sin(kz - \omega t)\underbrace{\left(k\sqrt{\frac{\epsilon}{\mu}} - \frac{\omega\epsilon}{c}\right)}_{=0}\hat{x}\right]$$

$$\boxed{\vec{J} = \frac{c}{4\pi} E_0 e^{-\alpha z}\, \alpha \cos(kz - \omega t)\,\hat{x}}$$

$k\sqrt{\frac{\epsilon}{\epsilon_0}}$ $\quad \frac{\omega\epsilon}{\epsilon_0\sqrt{\epsilon_0\mu_0}}$ $\quad \frac{\epsilon}{n} = \sqrt{\frac{\epsilon}{\mu}}$

$\frac{v}{c}$

b) If the solar wind has a mass density ρ, deduce the acceleration (time averaged, per unit volume) being exerted by the wave on the solar wind at a location $z > 0$.

→ Start with the Lorentz force law: $\vec{F} = q\left(\vec{E} + \frac{\vec{v}}{c}\vec{B}\right)$

thus the force at a point $\vec{r}$ is given by:

$$\vec{F}(\vec{r}) = e\rho(\vec{r})\left\{ E_0 e^{-\alpha z}\cos(kz-\omega t)\,\hat{x} + \frac{\vec{v}}{c} \times E_0 e^{-\alpha z}\cos(kz-\omega t)\,\hat{y} \right\}$$

find $\vec{v} = \frac{\vec{J}(r)}{\rho(r)} = \frac{1}{\rho(r)}\frac{c}{4\pi} E_0 e^{-\alpha z}\alpha\cos(kz-\omega t)\,\hat{x}$

$$F(r) = e\rho(r) E_0 e^{-\alpha z}\left[\cos(kz-\omega t)\,\hat{x} + \frac{\alpha}{4\pi\rho(r)} E_0 e^{-\alpha z}\cos^2(kz-\omega t)\,\hat{z}\right]$$

taking time averages:

$$\langle\cos(kz-\omega t)\rangle = 0 \quad , \quad \langle\cos^2(kz-\omega t)\rangle = \frac{1}{2}$$

thus $\boxed{\vec{F} = \underset{\text{charge}}{e} E_0^2 e^{-2\alpha z}\left(\frac{\alpha}{8\pi}\right)\hat{z}}$

huh?
what is this force?
what charge?