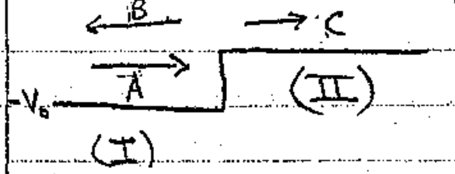
Spring 1997 #8

A one-dimensional potential $V(x) = \begin{cases} -V_0 & x<0 \\ 0 & x>0 \end{cases}$ can be used to describe a metal insulator junction. Electrons with an energy sufficient to leave the metal (the $x<0$ region) may be reflected at the interface ($x=0$).

a) Find the reflection coefficient at the surface for an electron with energy $E>0$



Solve Schrödinger's eq in regions (I), (II)

in Region (I):

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi - V_0\psi = E\psi$$

$$\frac{d^2}{dx^2}\psi = -\frac{2m(V_0+E)}{\hbar^2}\psi = -k_1^2\psi \qquad \left(k_1^2 = \frac{2m(V_0+E)}{\hbar^2}\right.$$

$$\psi_I = Ae^{ikx} + Be^{-ikx}$$

in Region (II)

$$\frac{d^2}{dx^2}\psi = -\frac{2mE}{\hbar^2} = -k_2^2\psi \qquad \left(k_2^2 = \frac{2mE}{\hbar^2}\right)$$

$$\psi_{II} = Ce^{ik_2x}$$

Now, find probability current density for each part of ψ

$$j = \frac{\hbar}{2mi}(\psi^*\nabla\psi - \psi\nabla\psi^*) \qquad \text{(Shankar eq 5.3.8}$$

for incident plane wave

$$j = \frac{\hbar}{2mi}\left(Ae^{-ik_1x}A(ik_1)e^{+ik_1x} - Ae^{ik_1x}A(-ik_1)e^{-ik_1x}\right)$$

$$j_A = \frac{\hbar}{2m\cancel{i}}(\cancel{2}\cancel{i}k_1)A^2 = \frac{|A|^2\hbar k_1}{m}$$

for reflected wave

$$j_B = \frac{\hbar}{2mi}\left(B\cancel{e^{ikx}}(-ik_1)\cancel{e^{-ikx}}B - B\cancel{e^{-ikx}}B(ik_1)\cancel{e^{ikx}}\right)$$

$$j_B = \frac{\hbar}{2mi}(-2ik_1)B^2 = \frac{-|B|^2\hbar k_1}{m}$$

$$j_c = \frac{|C|^2 \hbar k_2}{m}$$

Now we continuity of ψ, $\frac{d}{dx}\psi$ at $x=0$ to solve for B, C

$$A + B = C \qquad A(\cancel{i}k_1) - B(\cancel{i}k_1) = C(\cancel{i}k_2)$$

$$C = \frac{k_1(A-B)}{k_2}$$

$$k_2 A + k_2 B = k_1 A - k_1 B$$

$$\boxed{B = \frac{A(k_1 - k_2)}{(k_2 + k_1)}}$$

$$C = \frac{k_1}{k_2}\left(A - A\frac{(k_1-k_2)}{(k_2+k_1)}\right)$$

$$C = \frac{k_1 A}{k_2}\left(\frac{k_2 + \cancel{k_1} - \cancel{k_1} + k_2}{k_2 + k_1}\right) = \boxed{\frac{2A}{k_1 +}}$$

Reflection coefficient $R = \frac{|j_B|}{j_A} = \frac{|B|^2}{|A|^2}$

$$R = \frac{(k_1 - k_2)^2}{(k_2+k_1)^2} = \frac{k_1^2 - 2k_1k_2 + k_2^2}{k_2^2 + 2k_1k_2 + k_1^2} = \frac{E - 2\sqrt{E(E+V_0)} + E + V_0}{E + V_0 + 2\sqrt{E(E+V_0)} + E}$$

$$\boxed{R = \frac{2E + V_0 - 2\sqrt{E(E+V_0)}}{2E + V_0 + 2\sqrt{E(E+V_0)}}}$$

b) What is ratio of reflected to transmitted electrons for E=0.1eV, V

$$\frac{R}{T} = \left(\frac{(k_1-k_2)^2}{(k_2+k_1)^2}\right)\left(\frac{4k_1^2}{(k_1+k_2)^2}\frac{k_2}{k_1}\right)^{-1} = \frac{(k_1-k_2)^2}{4k_1k_2} = \frac{k_1^2 - 2k_1k_2 + k_2^2}{4k_1k_2}$$

$$\frac{R}{T} = \frac{2E - 2\sqrt{E(E+V_0)} + V_0}{4\sqrt{E(E+V_0)}} = \frac{10.2 - 2\sqrt{0.1(10.1)}}{4\sqrt{0.1(10.1)}}$$

$$\boxed{\frac{R}{T} \approx \frac{8.2}{4} = 2}$$ ← so roughly twice as many electro get reflected as get transmitted!

$$\begin{pmatrix} \varepsilon_1 & 2 \\ 0 & \varepsilon_2 \end{pmatrix}\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} = \begin{pmatrix} 0 & i\varepsilon_1 \\ i\varepsilon_2 & 0 \end{pmatrix}$$