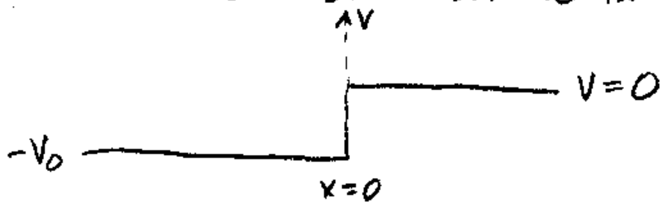
Spring 1997 # 8 (p 1 of 3)

A 1-D potential

$$V(x) = \begin{cases} -V_0 & x < 0 \\ 0 & x > 0 \end{cases}$$

Can be used to describe a metal-insulator junction. $V_0 > 0$ & constant. Electrons with an energy sufficient to leave the metal ($x < 0$ region) may be reflected at the interface ($x=0$). (See Zettili p 210-212)

(a) Find the reflection coefficient at the surface for an electron with energy $E > 0$.



need to solve Schrödinger's equation for $x<0$ and $x>0$.

$\underline{x<0}$

$$-\frac{1}{2m}\frac{d^2\psi_<}{dx^2} - V_0\psi_< = E\psi_< \Rightarrow \frac{d^2\psi_<}{dx^2} + K_<^2\psi_< = 0 \quad , K_<^2 = 2m(V_0+E)$$

So,

$$\psi_<(x) = A\,e^{iK_<x} + B\,e^{-iK_<x}$$

For a finite solution as $x \to -\infty$, we require $B=0$; but $K_<$ changes direction for a reflected wave, so,

$$\psi_<(x) = A_I e^{iK_<x} + B_R e^{-iK_<x} \quad , \begin{matrix} I \equiv \text{incident} \\ R = \text{reflected} \end{matrix}$$

$\underline{x>0}$

$$-\frac{1}{2m}\frac{d^2\psi_>}{dx^2} - E\psi_> = 0 \Rightarrow \frac{d^2\psi_>}{dx^2} + K_>^2\psi_> = 0 \quad , K_>^2 = 2mE$$

So,

$$\psi_>(x) = C\,e^{iK_>x} + D\,e^{-iK_>x}$$

Since $D\,e^{-iK_>x}$ represents waves in the $-x$ direction, we have

$$\psi_>(x) = C_T e^{+iK_>x} \quad , T = \text{Transmitted}$$

Thus, the complete wave function is thus

$$\Psi(x,t) = \begin{cases} A_I e^{i(K_<x-\omega t)} + B_R e^{-i(K_<x+\omega t)} & x<0 \\ C_T\,e^{i(K_>x-\omega t)} & x>0 \end{cases}$$

The reflection coefficient is defined as

$$R = \left| \frac{\text{reflected current density}}{\text{incident current density}} \right| = \left| \frac{J_{reflected}}{J_{incident}} \right| \quad (1)$$

where

$$J_{incident} = \frac{i}{2m}\left[\psi_i(x)\frac{d\psi_i^*(x)}{dx} - \psi_i^*(x)\frac{d\psi_i(x)}{dx}\right] \quad , \psi_i(x) = A_I e^{ik_< x}$$

$$= \frac{i}{2m}|A_I|^2\left[e^{ik_< x}(-ik_<)e^{-ik_< x} - e^{-ik_< x}(ik_<)e^{ik_< x}\right]$$

$$= \frac{i}{2m}|A_I|^2(-ik_<)\left[1+1\right] = \frac{k_<}{m}|A_I|^2 \quad (2)$$

and likewise, we have

$$J_{reflected} = -\frac{k_<}{m}|B_R|^2 \quad (3)$$

So, substituting eq.s (3) and (2) into eq (1) yields

$$R = \left| \frac{\cancel{\frac{k_<}{m}}}{\cancel{\frac{k_<}{m}}} \frac{|B_R|^2}{|A_I|^2} \right| = \frac{|B_R|^2}{|A_I|^2} \quad (4)$$

Now, we just need to determine A_I and B_R. We do this via B.C.s

at $x=0$,

$$A_I + B_R = C_T$$

and

$$ik_< A_I + -ik_< B_R = ik_> C_T \Rightarrow C_T = \frac{k_<}{k_>}(A_I - B_R)$$

Solving each for C_T and writing B_R in terms of A_I yields

$$A_I + B_R = \frac{k_<}{k_>}A_I - \frac{k_<}{k_>}B_R$$

$$\Rightarrow \quad A_I\left(1-\frac{K_<}{K_>}\right) = -B_R\left(1+\frac{K_<}{K_>}\right)$$

$$\therefore \quad B_R = -\frac{K_> - K_<}{K_> + K_<} A_I = \frac{K_< - K_>}{K_> + K_<} A_I \qquad (5)$$

Substituting this result into eq (4) yields

$$R = \left|\frac{K_< - K_>}{K_> + K_<}\right|^2 = \frac{K_<^2 + K_>^2 - 2K_<K_>}{K_<^2 + K_>^2 + 2K_>K_<}$$

$$= \frac{2mV_0 + 2mE + 2mE - 2\sqrt{(2mE)(2mE+2mV_0)}}{2mV_0 + 2mE + 2mE + 2\sqrt{(2mE)(2mE+2mV_0)}}$$

$$\therefore \boxed{R = \frac{2E + V_0 - 2\sqrt{E(E+V_0)}}{2E + V_0 + 2\sqrt{E(E+V_0)}}}$$

(b) what is the ratio of the number of the reflected electrons to the number of transmitted electrons for a junction of $V_0 = 10\,eV$ and electrons with an energy of $0.1\,eV$?

$$R = \frac{0.2\,eV + 10\,eV - 2eV\sqrt{0.1(10.1)}}{0.2\,eV + 10\,eV + 2eV\sqrt{0.1(10.1)}} \simeq \frac{10.2\,eV - 2eV\sqrt{1.01}}{10.2\,eV + 2eV\sqrt{1.01}}$$

$$\simeq \frac{8.2\,eV}{12.2\,eV} \simeq \frac{2}{3}$$

$$T = 1 - \frac{2}{3} = \frac{1}{3}$$

Thus, $\boxed{\frac{R}{T} \simeq \frac{\frac{2}{3}}{\frac{1}{3}} = 2}$