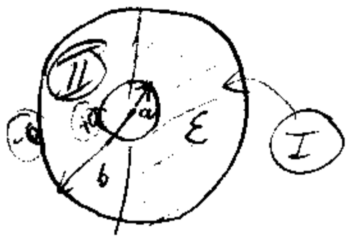
F98 #1



2 concentric conductors

$\vec{E}$ must be radial (since Φ is radial)

and since $\Phi \sim \frac{1}{r}$ (see mode expansion) $E \sim \frac{1}{r^2}\hat{r}$

then by gauss' law: (I) $\cancel{4}2\pi r^2 \vec{D} = \sigma_I \, 2\pi a^2$

$$\Rightarrow \vec{E} = \frac{a^2 \sigma_I}{\varepsilon r^2} \hat{r}$$

check units: $\vec{\nabla} \cdot \vec{D} = \frac{1}{r^2 \sin\theta} \cancel{\frac{\partial}{\partial r} \frac{a^2 \sigma_I}{r^2}} r^2 \sin\theta + \cancel{\frac{1}{r^2 \sin\theta} \frac{\partial}{\partial\theta}}$?
0

likewise, in II $\vec{E} = \frac{a^2 \sigma_{II}}{\varepsilon_0 r^2} \hat{r}$

and the tangential component at the boundary is continuous, so:

$$\frac{\sigma_I}{\varepsilon} = \frac{\sigma_{II}}{\varepsilon_0}$$

and the total charge is $+Q$, so $2\pi a^2 \sigma_I + 2\pi a^2 \frac{\varepsilon_0}{\varepsilon} \sigma_I = Q$

$\rightarrow \sigma_I = \boxed{\frac{Q}{2\pi a^2 (1 + \frac{\varepsilon_0}{\varepsilon})}} \Rightarrow \boxed{\sigma_{II} = \frac{\varepsilon}{\varepsilon_0} \sigma_I}$

then $\vec{E}_I = \frac{a^2}{\varepsilon r^2} \frac{\varepsilon}{\varepsilon_0} \frac{Q}{2\pi a^2 (1 + \frac{\varepsilon_0}{\varepsilon})} \hat{r}$

$$\vec{E}_{II} = \frac{a^2}{\varepsilon_0 r^2} \frac{Q}{2\pi a^2 (1 + \frac{\varepsilon_0}{\varepsilon})} \hat{r} = \vec{E}_I$$

(the fields are equal

Polarization: $\sigma_{Bound} = \vec{P} \cdot \hat{n} = P_r$; $P \sim D - E \sim (\varepsilon - 1) E$