Fall 1998 # 3

$$\vec{M}(B) = \frac{M_0 \vec{B}}{B_c + B}$$

First, calculate field of magnetized sphere without applied field

$$\nabla \times \vec{H} = \frac{4\pi}{c} J_f + \frac{1}{c}\frac{\partial D}{\partial t}$$

$$\Rightarrow \nabla \times \vec{H} = 0 \qquad \Rightarrow \vec{H} = -\nabla \Phi_M$$

$$\Phi_m = \sum_{\ell=0}^{\infty} \left[A_\ell r^\ell + \frac{B_\ell}{r^{\ell+1}}\right] P_\ell(\cos\theta)$$ ← since object is a sphere

$$r > R_0 \quad = \sum_{\ell=0}^{\infty} \frac{B_\ell}{r^{\ell+1}} P_\ell \cos\theta$$

In sphere $\vec{H} = \vec{B}_i - 4\pi \vec{M}_i$ for $\vec{B}$ parallel to $\vec{M}$

Boundry conditions say $B_r, H_\theta$ are continous

$$\vec{B}_{in} = B_i \hat{z} = B_i(\hat{r}\cos\theta - \hat{\theta}\sin\theta)$$

$$H_{in} = (B_i - 4\pi M_i)\hat{z} = (B_i - 4\pi M_i)(\hat{r}\cos\theta - \hat{\theta}\sin\theta)$$

outside $\vec{B} = \vec{H}$

$$\vec{B}_{out} = \vec{H}_{out} = -\nabla\Phi_m = \frac{-\partial \Phi_m}{\partial r}\hat{r} - \frac{1}{r}\frac{\partial \Phi_m}{\partial \theta}\hat{\theta}$$

$$= \left[(\ell+1) B_\ell r^{-(\ell+2)} P_\ell(\cos\theta)\right]\hat{r} - \left[B_\ell e^{-(\ell+2)} \frac{d}{d\theta} P_\ell(\cos\theta)\right]\hat{\theta}$$

for $B_\perp$ continous to be true $\ell$ must equal 1

$$\vec{B}_{out} = \vec{H}_{out} = \left[2 B_1 r^{-3}\cos\theta\right]\hat{r} + \left[B_1 r^{-3}\sin\theta\right]\hat{\theta}$$

Applying $B_\perp$ continous gives

$$B_i = \frac{2B}{R_o^3}$$

And $H_\parallel$ continous

$$(B_i - 4\pi M_i) = \frac{-B}{R_o^3} \qquad \frac{2c}{R_o^3} - 4\pi M_i^o = \frac{-c}{R_o^3} \qquad c = \frac{4\pi}{3} M_i R_o^3$$

But we ~~know~~ Know $M$ is there because of $B$ applied $\Rightarrow B_o$

$$\Rightarrow \vec{B_i} = B_o + \frac{8\pi M_i}{3} \qquad H_i = -\frac{4\pi M_i}{3} + B_o$$

Then

$$\vec{B}_{in} = B_o \left(1 + \frac{8\pi M_o}{3(B_c + B_o)}\right)(\cos\theta \hat{r} - \sin\theta \hat{\theta})$$

$$H_{in} = B_o \left(1 - \frac{4\pi M_o}{3(B_c + B_o)}\right)(\cos\theta \hat{r} - \sin\theta \hat{\theta})$$

$$\bar{B}_{out} = \bar{H}_{out} = \left[B_o + \frac{8\pi M_o R_o^3}{(B_c + B_o) r^3}\right]\cos\theta \hat{r} + \left[B_o + \frac{4\pi}{3} \frac{M_o R_o^3 B_o}{(B_c + B_o) r^3}\right]\sin\theta \hat{\theta}$$

$$H_{in} = B_o \left(1 - \frac{4\pi M_o}{3(B_c + B_o)}\right) = B_o \left[1 - \left(\frac{8\pi M_o}{3 B_c}\right)\left(\frac{1}{2(1 + B_o/B_c)}\right)\right]$$

$<1$

if $\frac{8\pi M_o}{3B} > 1$ It takes a large field to have $\bar{H}$ be parallel to applied $B$ field