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$\vec{B} = (B\sin\theta,\ 0,\ B\cos\theta)$ $\qquad t=0$ spin z $= \hbar/2 = \frac{1}{2}$ with $\hbar = 1$

$$H = -\vec{\mu}\cdot\vec{B} = \frac{g\mu_B \vec{B}\cdot\sigma}{2} \qquad g = 2 \Rightarrow H = \mu_B \vec{B}\cdot\vec{\sigma}$$

$$H = \mu_0 \begin{pmatrix} B\cos\theta & B\sin\theta \\ B\sin\theta & -B\cos\theta \end{pmatrix} = \frac{\omega}{2}\hat{n}\cdot\sigma \qquad \omega = 2\mu_B B$$

$$\psi(t) = e^{-iHt}\psi(0) = e^{-i\omega t\, \hat{n}\cdot\sigma/2}\psi(0)$$

eigenstates of $\hat{n}\cdot\vec{\sigma}$, in σ_z basis are

$$\chi_+ = \begin{pmatrix} \cos\theta/2 \\ \sin\theta/2 \end{pmatrix} \qquad \chi_- = \begin{pmatrix} \sin\frac{\theta}{2} \\ -\cos\theta/2 \end{pmatrix} \quad \text{eiganvalues } \pm 1$$

$$\psi(t) = C_+ e^{-i\omega t/2} + C_- e^{i\omega t/2}\chi_- = \begin{pmatrix} C_+ e^{-i\omega t/2}\cos\frac{\theta}{2} + C_- e^{i\omega t/2}\sin\frac{\theta}{2} \\ C_+ e^{-i\omega t/2}\sin\frac{\theta}{2} - C_- e^{i\omega t}\cos\frac{\theta}{2} \end{pmatrix}$$

So, since $\psi(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$C_+\sin\frac{\theta}{2} = C_-\cos\theta/2 \qquad C_+ = C_-\frac{\cos\frac{\theta}{2}}{\sin\theta/2} \qquad C_+\cos\frac{\theta}{2} + C_-\sin\frac{\theta}{2} = 1$$

$$C_-\cos^2\frac{\theta}{2} + C_-\sin^2\theta/2 = \sin\theta/2$$

$$C_- = \sin\theta/2$$

$$\psi(t) = \begin{pmatrix} e^{-i\omega t/2}\cos^2\frac{\theta}{2} + e^{i\omega t/2}\sin^2\frac{\theta}{2} \\ e^{-i\omega t/2}\cos\frac{\theta}{2}\sin\frac{\theta}{2} - e^{i\omega t}\sin\frac{\theta}{2}\cos\theta/2 \end{pmatrix} \qquad C_+ = \cos\theta/2 \qquad = \begin{pmatrix} a_+(t) \\ b_-(t) \end{pmatrix}$$

Probability of being in spin up state $|a_+(t)|^2$

since $P_+ = |\langle z_+|\psi(t)\rangle|^2 \qquad z_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$a_+(t) = e^{-i\omega t/2}\cos^2\frac{\theta}{2} + e^{i\omega t/2}\sin^2\theta/2$$

$$= e^{-i\omega t/2}\cos^2\theta/2 + e^{i\omega t/2} - e^{i\omega t/2}\cos^2\theta/2$$

$$= \cos^2\frac{\theta}{2}\left(e^{-i\omega t/2} - e^{i\omega t/2}\right) + e^{i\omega t/2}$$

$$= -2i\cos^2\frac{\theta}{2}\sin\frac{\omega t}{2} + \cos\frac{\omega t}{2} + i\sin\frac{\omega t}{2}$$

$\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}}$ $$= -i(1+\cos\theta)\sin\frac{\omega t}{2} + \cos\frac{\omega t}{2} + \cancel{i\sin\frac{\omega t}{2}}$$

$$= \cos\frac{\omega t}{2} - i\cos\theta\sin\frac{\omega t}{2}$$

$$|a_+(t)|^2 = \cos^2\frac{\omega t}{2} + \cos^2\theta\sin^2\frac{\omega t}{2}$$

at $\theta = 0$ probability is 1

at $\theta = \frac{\pi}{2}$ probability is $\cos^2\frac{\omega t}{2}$