p105 Spring 1998 # 13

A neutron in 3D (mass M) scatters off a very heavy nucleus, and the force between them is given by the Yukawa potential

$$V(r) = V_0 \frac{e^{-\mu r}}{\mu r}$$

What is the Born approximation for the differential cross section, as a function of the scattering angle Θ and the neutron's initial momentum $\hbar k$?

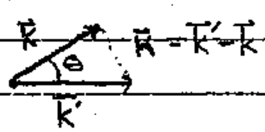
→ Differential cross section given by:

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 \qquad \text{(Griffith's QM eq 11.14)}$$

the Born approximation of the scattering amplitude for spherically symmetric potentials is:

$$f(\theta) \cong -\frac{2m}{\hbar^2 K}\int_0^\infty r\,V(r)\sin(Kr)\,dr \qquad \text{(Griffith's QM eq 11.78)}$$

where $\vec{K} = \vec{k}' - \vec{k}$ = (scattered − incident) wave vector



since $k' = k$ the law of cosines yields
$K^2 = 2k^2 - 2k^2\cos\Theta$

$$K = k\sqrt{2(1-\cos\Theta)} = 2k\sin(\Theta/2)$$

$$f(\theta) \cong -\frac{2m}{\hbar^2 K}\int_0^\infty \frac{V_0}{\mu} e^{-\mu r}\sin(Kr)\,dr$$

recall $\sin(Kr) = \frac{1}{2i}\left(e^{iKr} - e^{-iKr}\right)$

$$f(\theta) \cong -\frac{2mV_0}{2\hbar^2 K i\mu}\int_0^\infty \left(e^{ikr-\mu r} - e^{-ikr-\mu r}\right)dr$$

$$= \frac{-mV_0}{\mu\hbar^2 K i}\left[\left[\frac{1}{ik-\mu}e^{(ik-\mu)r}\right]_0^\infty - \frac{1}{-ik-\mu}\left[e^{(-ik-\mu)r}\right]_0^\infty\right]$$

$$f(\theta) = \frac{-mV_0}{\mu\hbar^2 Ki}\left(\frac{-1}{iK-\mu} + \frac{-1}{iK+\mu}\right) = \frac{+mV_0}{\mu\hbar^2 Ki}\left(\frac{iK+\cancel{\mu}+iK-\cancel{\mu}}{-K^2-\mu^2}\right)$$

$$f(\theta) = -\frac{2mV_0}{\mu\hbar^2(K^2+\mu^2)}$$

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 = \frac{4m^2V_0^2}{\mu^2\hbar^4(K^2+\mu^2)^2}$$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{4m^2V_0^2}{\mu^2\hbar^4(4k^2\sin^2(\frac{\theta}{2})+\mu^2)^2}}$$