note: $\sin^2\theta = \frac{1}{2}[1-\cos 2\theta]$

Spring 1998 #13

A neutron in 3D (mass M) scatters off a very heavy nucleus, and the force between them is given by the Yukawa potential:

$$V(r) = V_0 \frac{e^{-\mu r}}{\mu r}$$

what is the Born approximation for the differential cross section, as a function of the scattering angle θ and the neutron's initial momentum $\hbar k$?

set $\hbar = 1$:

the differential cross section for the Born approximation is given by

$$\frac{d\sigma}{d\Omega} = |f(\theta,\phi)|^2 \quad \text{(Abers eq 8.22)}$$

, where for spherical symmetry like the Yukawa potential, (Abers eq 8.40)

$$f(\theta,\phi) = -\frac{1}{q}\int_0^\infty r\sin(qr)\,U(r)\,dr \qquad , U(r) = 2mV(r)$$

$$= \frac{-2mV_0}{q}\int_0^\infty \frac{r}{\mu r} e^{-\mu r}\sin(qr)\,dr$$

$$= \frac{-2mV_0}{\mu q}\int_0^\infty \sin(qr)e^{-\mu r}dr = \frac{-2mV_0}{2\mu q i}\int_0^\infty e^{-\mu r}\left[e^{iqr} - e^{-iqr}\right]dr$$

$$= \frac{imV_0}{\mu q}\int_0^\infty \left[e^{-r(\mu-iq)} - e^{-r(\mu+iq)}\right]dr$$

$$= \frac{imV_0}{\mu q}\left\{\frac{-1}{\mu-iq}e^{-r(\mu-iq)} + \frac{1}{\mu+iq}e^{-r(\mu+iq)}\right\}_0^\infty$$

$$= \frac{imV_0}{\mu q}\left[\frac{1}{\mu+iq} - \frac{1}{\mu-iq}\right] = \frac{imV_0}{\mu q}\left[\frac{\mu-iq-\mu-iq}{\mu^2+q^2}\right]$$

$$= \frac{+2mV_0}{\mu}\frac{1}{\mu^2+q^2}$$

where momentum transfer, q, is

$$\vec{q} = \vec{k} - \vec{k}' \quad , |\vec{k}| = |\vec{k}'|$$

$$\Rightarrow q^2 = |\vec{k}|^2 + |\vec{k}'|^2 - 2\vec{k}\cdot\vec{k}'$$

$$= 2k^2 - 2k^2\cos\theta$$

$$= 2k^2[1-\cos\theta]$$

$$\boxed{q^2 = 4k^2\sin^2\left(\frac{\theta}{2}\right)}$$

So,

$$\boxed{\frac{d\sigma}{d\Omega} = |f(\theta,\phi)|^2 = \frac{4m^2V_0^2}{\mu^2}\frac{1}{(\mu^2+q^2)^2}}$$