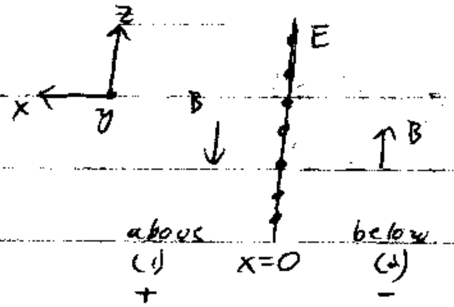
$$\vec{J} = J_0 \cos(\omega t)\, \delta(x)\, \hat{y} \qquad \text{why not call it } \vec{K}?!?$$

$$\underset{\left[\frac{A}{m}\right]}{\vec{J}} \quad \underset{[A]}{J_0} \quad \underset{[1/m]}{\delta(x)}$$



a) from Griffiths (3^{rd}) p. 396

$$\epsilon_1 E_1^{\perp} - \epsilon_2 E_2^{\perp} = \sigma_f \qquad B_1^{\perp} - B_2^{\perp} = 0$$

$$\vec{E}_1^{\parallel} - \vec{E}_2^{\parallel} = 0 \qquad \frac{1}{\mu_1}\vec{B}_1^{\parallel} - \frac{1}{\mu_2}\vec{B}_2^{\parallel} = \vec{K}_f \times \hat{n}$$

now we assume we are dealing with free space so $\epsilon_1 = \epsilon_2 = \epsilon_0$ and $\mu_1 = \mu_2 = \mu_0$

As the sheet is neutral $\sigma_f = 0$. As for $\hat{n}$ it is defined to go from "below" to "above" – so in the +ve x direction.

Hence:

$$\vec{K}_f \times \hat{n} = J_0 \cos(\omega t)\delta(x) \overbrace{[\hat{y} \times \hat{x}]}^{-\hat{z}} = -J_0 \cos(\omega t)\delta(x)\,\hat{z}$$

so in summary:

$$\left.\begin{array}{l} E_1^{\perp} - E_2^{\perp} = 0 \\ \vec{E}_1^{\parallel} - \vec{E}_2^{\parallel} = 0 \end{array}\right\} \Rightarrow \vec{E}_1 = \vec{E}_2 \equiv E_0 \qquad \begin{array}{l} B_1^{\perp} - B_2^{\perp} = 0 \\ \vec{B}_1^{\parallel} - \vec{B}_2^{\parallel} = -\mu_0 J_0 \cos(\omega t)\delta(z)\,\hat{z} \end{array}$$

b) From Maxwell's equations: (in free space)

(i) $\vec{\nabla}\cdot\vec{E}=0$ $\quad\quad$ $\vec{\nabla}\cdot\vec{B}=0$ (iii)

(ii) $\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$ $\quad\quad$ $\vec{\nabla}\times\vec{B}=\mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$ (iv)

from (ii): $\vec{\nabla}\times(\vec{\nabla}\times\vec{E})=\vec{\nabla}(\underbrace{\vec{\nabla}\cdot\vec{E}}_{=0})-\nabla^2 E=-\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$

$$=-\mu_0\epsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$$

so $\nabla^2 E=\underbrace{\mu_0\epsilon_0}_{=\frac{1}{c^2}}\frac{\partial^2 E}{\partial t^2}=\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$

similarly for (iv): $\vec{\nabla}\times(\vec{\nabla}\times\vec{B})=\vec{\nabla}(\underbrace{\vec{\nabla}\cdot\vec{B}}_{=0})-\nabla^2 B=\mu_0\epsilon_0\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{E})$

$$=-\mu_0\epsilon_0\frac{\partial^2 B}{\partial t^2}$$

$$\Rightarrow\quad \nabla^2 B=\frac{1}{c^2}\frac{\partial^2 B}{\partial t^2}$$

for $x>0$: $\vec{E}(x,t)=E_+\cos(kx-\omega t)\,\hat{y}$

$\vec{B}(x,t)=-B_+\cos(kx-\omega t)\,\hat{z}$

$x<0$ $\quad$ $\vec{E}(x,t)=E_-\cos(kx-\omega t)\,\hat{y}$

$\vec{B}(x,t)=B_-\cos(kx-\omega t)\,\hat{z}$

from the boundary conditions $E_+=E_-=E_0$

$$-B_+\cos(kx-\omega t)-B_-\cos(kx-\omega t)=-J_0\mu_0\cos(\omega t)\delta(x)$$

for $x=0$

$$-(B_++B_-)\cancel{\cos(\omega t)}=-J_0'\mu_0\cancel{\cos(\omega t)}$$

$$\Rightarrow\quad B_++B_-=J_0'\mu_0$$

now there is the relation:

$$\vec{B} = \frac{1}{c}(\hat{k} \times \underbrace{\vec{E}}_{E(t)\hat{y}})\ ; \quad \hat{k} = \hat{x}$$

$$= \frac{1}{c} E(t)\,\hat{y}$$

so $\quad B_+ \cancel{\cos(kx-\omega t)} = E_+ \cancel{\cos(kx-\omega t)}$

$\quad B_- \cancel{\cos(kx+\omega t)} = E_- \cancel{\cos(kx-\omega t)}$

$\Rightarrow \quad B_+ = E_+\ ; \ B_- = E_- \quad$ but $\ E_+ = E_- \ \Rightarrow \ B_+ = B_- = B_0$

hence

$$2B_0 = J_0' \mu_0$$

$\Rightarrow \quad B_0 = \frac{J_0'\mu_0}{2} \ \Rightarrow \ E_0 = \frac{J_0'\mu_0 c}{2}$

so $\quad x > 0$

$$\vec{E}(x,t) = \frac{J_0'\mu_0 c}{2}\cos(kx-\omega t)\,\hat{y}$$

$$\vec{B}(x,t) = -\frac{J_0'\mu_0}{2}\cos(kx-\omega t)\,\hat{z}$$

$x < 0 \quad \vec{E}(x,t) = \frac{J_0'\mu_0 c}{2}\cos(kx-\omega t)\,\hat{y}$

$$\vec{B}(x,t) = \frac{J_0'\mu_0}{2}\cos(kx-\omega t)\,\hat{z}$$

c) Energy flux $(x>0)$: $\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B}) = \frac{1}{\mu_0}\left(\frac{J_0'^2\mu_0^2 c}{4}\cos^2(kx-\omega t)\right)\left[\underbrace{\hat{y}\times(-\hat{z})}_{=-\hat{x}}\right]$?!?!?

so $\quad \vec{S} = -\frac{J_0'^2\mu_0 c}{4}\cos^2(kx-\omega t)$

• time averaged radiated power:

$$\langle S \rangle = -\frac{J_0'^2 \mu_0 c}{4} \langle \underbrace{\cos^2(kx-\omega t)}_{=\frac{1}{2}} \rangle = -\frac{J_0'^2 \mu_0 c}{8}$$

• Work done by E-field: (per unit area)

from Griffith (3^{rd}) p. 346

$$\overset{3D}{\frac{dW}{dt} = \int_V (\vec{E}\cdot\vec{J})\,d\tau} \Rightarrow \overset{2D}{\frac{dW}{dt} = \int_A (\vec{E}\cdot\vec{K})\,da}$$

so $$W = \int \vec{E}\cdot\vec{K}\,dt = \int \frac{J_0'\mu_0 c}{2}\cos(kx-\omega t)\,J_0\cos(\omega t)\,\delta(x)\,dt \qquad x=0$$

$$= \frac{J_0'^2 \mu_0 c}{2} \underbrace{\int \cos^2(\omega t)\,dt}_{=\frac{1}{2}}$$

$$= \frac{J_0'^2 \mu_0 c}{4}$$