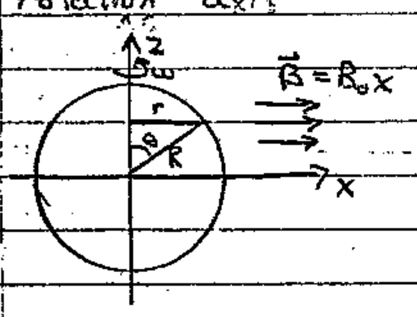
A conducting sphere of radius R is charged to potential V an is rotated about a diameter at an angular velocity ω. A unifor magnetic field B is applied at right angles to the rotation ax Find the torque on the sphere.

In general, the torque is given by $\vec{\tau} = \vec{m} \times \vec{B}$ where $\vec{m}$ is the magnetic dipole moment. So if one can find $\vec{m}$, the torque is easily obtained. The vector $\vec{m}$ is given by:

$$\vec{m} = \frac{1}{2c}\int \vec{r}' \times J(\vec{r}')\, d^3r' \qquad \text{(Greiner eq 10.7)}$$

Since the sphere has potential V, we know its charge can be found from $V = Q/R \Rightarrow Q = VR$. The surface charge density is thus $\sigma = \frac{Q}{4\pi R^2} = V/4\pi R$. Now to find the c use $\vec{J} = \sigma\vec{v}$, where $\vec{v}$ is the linear velocity. The vector $\vec{v}$ given by $\vec{v} = \vec{\omega} r$, where r is the distance from the rotation axis



from the diagram $r = R\sin\theta$, which gives $|v| = \omega R \sin\theta$ and thus

$$\boxed{\vec{J} = \frac{\text{Voltage}}{4\pi}\,\omega \sin\theta\, \delta(r-R)\, \hat{\phi}}$$

integrating to find $\vec{m}$ yields

$$\vec{m} = \frac{\hat{z}}{2c}\int \underset{\uparrow}{r}\, \frac{\text{Volt}}{4\pi}\,\omega\sin\theta\,\delta(r-R)\, r^2 \sin\theta\, dr\, d\theta\, d\phi$$

this r is different from $r = R\sin\theta$

$$\vec{m} = \left(\frac{\hat{z}}{2c}\right)\left(\frac{V\omega}{4\pi}\right) R^3\, 2\pi \int_0^\pi \sin^2\theta\, d\theta$$

$$= \frac{\hat{z}V\omega R^3}{4c}\int \frac{1}{2}(1-\cos(2\theta))\, d\theta$$

$$= \frac{\hat{z}V\omega R^3}{4c}\left[\frac{1}{2}\theta - \frac{1}{4}\sin(2\theta)\right]_0^\pi \Rightarrow \boxed{\vec{m} = \frac{V\omega R^3 \pi \hat{z}}{8c}}$$

For $\vec{B} = B_0\hat{x}$ the torque is:

$$\vec{\tau} = \vec{m} \times \vec{B} = \frac{V\omega R^3 \pi B}{8c} (\hat{z} \times \hat{x})$$

$$\boxed{\tau = \frac{V\omega R^3 \pi B}{8c} \hat{y}}$$