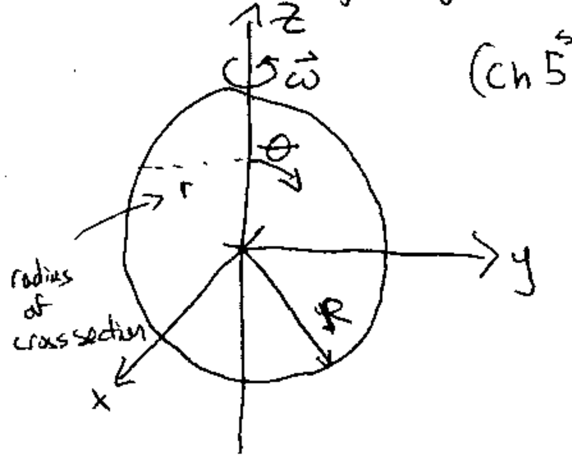
Spring 1998 #5

A conducting sphere of radius R is charged to potential V and is rotated about a diameter at an angular velocity ω. A uniform magnetic field B is applied at right angles to the rotation axis. Find the torque on the sphere.



(see Ch 5 Jackson)

torque: $\vec{\tau} = \vec{m} \times \vec{B}$, $\vec{m} = \frac{1}{2}\int \vec{r} \times \vec{J}(r)\, d^3r$

& $\vec{J} = \sigma \vec{v}$, $\vec{v} = \vec{\omega} r$

So,

$$\vec{J} = \sigma \omega r \quad , \quad \sigma = \frac{Q_{total}}{Area} = \frac{Q_{total}}{4\pi R^2} \quad (1)$$

we want to write J in terms of what is given

note: $V = \dfrac{Q_{tot}}{4\pi\varepsilon_0 R} \Rightarrow Q_{total} = 4\pi\varepsilon_0 R V \quad (2)$

combing eq.s (1) & (2) in our expression for J, we get

$$J = \frac{4\pi\varepsilon_0 R}{4\pi R^2}\omega r V = \frac{\varepsilon_0 \omega r V}{R}\delta(r'-R)$$

now note: $\dfrac{r}{R} = \sin\theta$

So, $\boxed{\vec{J} = \varepsilon_0 \omega V \sin\theta\, \delta(r'-R)\hat{\phi}}$

Substitute this result into or expression for $\vec{m}$

$$\vec{m} = \frac{1}{2}\int r' \varepsilon_0 \omega V \sin\theta\, \delta(r'-R)\, \sin\theta\, r^2 \sin\theta\, dr'\, d\theta\, d\phi = \boxed{\frac{4}{3}\varepsilon_0 \omega V \pi R^3 \hat{z}}$$

let $\vec{B} = B\hat{x} + B\hat{y}$

so, $\boxed{\vec{\tau} = \frac{4\pi}{3}\varepsilon_0 \omega V B R^3 \left(-\hat{x} + \hat{y}\right)}$