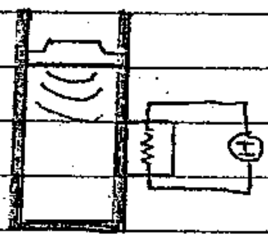
A pipe 0.1 m in diameter is filled with oxygen gas at 300K and suspended with wires in a vacuum chamber. The top end of the pipe is sealed with a piston that slides without friction. The piston has mounted on it a loudspeaker, and the total mass of piston + speaker is 2 kg. In equilibrium the lowest resonant frequency of the pipe is 90 Hz. If a current of 1 Amp is passed through a 10 Ω resistor attached to the pipe for 10 sec, what is the new resonant frequency?



Let's begin by finding the initial volume, assuming V_s is the constant speed of sound and that the gas can be considered ideal

the lowest resonance occurs when the length of the tube equals half the wavelength $h = \frac{\lambda}{2}$. Now use $\lambda = V_s/f$ to get $h = V_s/2f$. Thus the initial volume is $V = \pi r^2 h = \pi (.05)^2 V_s/2f$.
Now, the number of moles present can be found from

$$PV = \nu RT \qquad \Rightarrow \left(\frac{\cancel{2}\,kg}{\cancel{\pi(.05)^2}}\right)\left(\frac{\cancel{\pi(.05)^2}\,V_s}{\cancel{2}f}\right) = \nu R(300)$$

$$\nu = \frac{V_s}{300\,Rf}$$

To find the effect of the added energy, first consider the piston to be fixed when the current flows through the resistor.

$$W = I^2 R\,(\Delta t) = (0.1)^2(10)(10)\text{ Joule} = 1\text{ Joule}$$

Now, assuming the heat capacity for the ideal gas, $c_v = \frac{3}{2}R$ one can find the temperature change of the gas

$$\nu\,\tfrac{3}{2}R\,(\Delta T) = \Delta Q$$

$$\frac{V_s}{300\,Rf}\left(\tfrac{3}{2}R\right)\Delta T = 1\text{ Joule} \qquad \Rightarrow \Delta T = \frac{200\,f}{V_s}$$

thus the change in pressure is $(\Delta P)\,V = \nu R(\Delta T)$ $\Rightarrow$

$$\Delta P = \frac{\nu R \Delta T}{V} = \left(\frac{\cancel{v_s}}{300\cancel{R}\cancel{f}}\right)(\cancel{R})\left(\frac{\cancel{200}\cancel{f}}{\cancel{v_s}}\right)\left(\frac{2f}{\pi(.05)^2 v_s}\right) = \frac{4}{3}\frac{f}{\pi(.05)^2 v_s}$$

thus p now equals $\left(\frac{2}{(\pi(.05)^2)}\right) + \left(\frac{4}{3}\frac{f}{\pi(.05)^2 v_s}\right) = \frac{6v_s + 4f}{3v_s \pi(.05)^2}$

Now if the piston is released, the gas will undergo adiabatic expansion, where $pV^{\gamma} = \text{constant}$ $\quad (\gamma = \frac{C_p}{C_v} = 5/3)$

So, the pressure and volume satisfy

$$pV^{\gamma} = \left(\frac{6v_s + 4f}{3v_s \pi(.05)^2}\right)\left(\frac{\pi(.05)^2 v_s}{2f}\right)^{5/3} = P_{final} V_{final}^{5/3}$$

$$\left(\frac{6v_s + 4f_1}{3v_s \pi(.05)^2}\right)\left(\frac{\pi(.05)^2 v_s}{2f_1}\right)^{5/3} = \left(\frac{2}{\pi(.05)^2}\right)\left(\frac{\pi(.05)^2 v_s}{2f_2}\right)^{5/3}$$

where $\frac{\pi(.05)^2 v_s}{2f_2}$ is the final volume in terms of the new resonant frequency

$$\left(\frac{6v_s + 4f_1}{3v_s}\right)\left(f_1\right)^{-5/3} = (2)\left(f_2\right)^{-5/3}$$

$$(f_2)^{5/3} = \left(\frac{3v_s}{3v_s + 2f_1}\right)(f_1)^{5/3}$$

$$\boxed{f_2 = f_1\left(\frac{3v_s}{3v_s + 2f_1}\right)^{3/5}}$$ the new frequency will be lower

assuming $v_s \approx 340$ m/s and using $f_1 = 90$ Hz yields

using a calculator gives $\boxed{f_2 = 81.6 \text{ Hz}}$