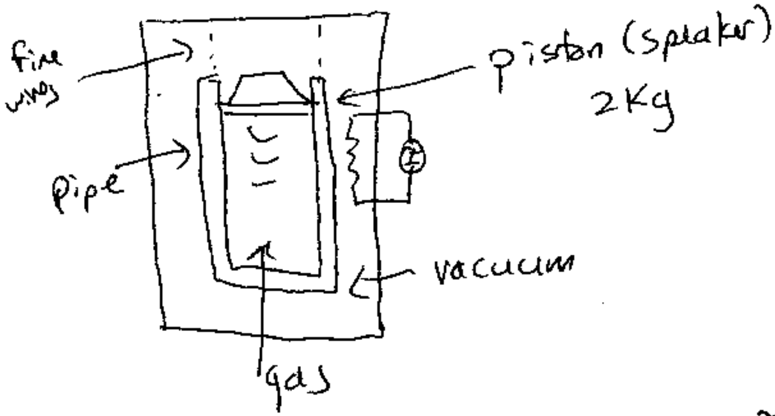
Spring 1998 #7 (p 1 of 2)



$O_2 \rightarrow$ diatomic ideal gas

$$C_v = \frac{5}{2}R \, , C_p = \frac{7}{2}R \Rightarrow \frac{C_p}{C_v} = \frac{7}{5} \equiv \gamma$$

$$\left.\begin{matrix} P_1 V_1^{\gamma} = P_2 V_2^{\gamma} \, , & A_1 = A_2 \end{matrix}\right\} \Rightarrow P_1 h_1^{\gamma} = P_2 h_2^{\gamma}$$

note: for lowest resonance ; $h = \frac{\lambda}{2}$, $\lambda = \frac{v_s}{f}$ ← speed of sound

So,

$$P_1 f_2^{\gamma} = P_2 f_1^{\gamma} \Rightarrow f_2 = f_1 \left(\frac{P_2}{P_1}\right)^{1/\gamma} = f_1 \left(\frac{P_2}{P_1}\right)^{5/7}$$

we are given that $f_1 = 90\ \text{Hz}$

we know that $P_1 = \frac{F}{A} = \frac{2\,\text{Kg} \cdot g}{\pi (.05\text{m})^2}$

$$P_2 = P_1 + \Delta P \quad , \Delta P = \frac{nR\Delta T}{V} \qquad (1)$$

where Energy given to gas $= I^2 R \Delta t = (0.1A)^2 (10\Omega)(10s) = 1J$

& $C_v = \frac{1}{n}\left(\frac{\Delta E}{\Delta T}\right)_V \Rightarrow n\frac{5}{2}R\Delta T = \Delta E$

$$\Rightarrow \Delta T = \frac{2}{5}\frac{1}{nR} \qquad (2)$$

substituting eq (2) into eq (1) we get

$$\Delta P = \frac{nR}{V}\frac{2}{5nR} = \frac{2}{5V}$$

So,

$$P_2 = P_1 + \frac{2}{5V} = \frac{F}{A} + \frac{2}{5V}$$ [redacted]

Thus,

$$f_2 = f_1 \left(\frac{P_2}{P_1} \right)^{5/7}$$

where $f_1 = 90\ Hz$, $P_1 = \frac{F}{A}$, $P_2 = \frac{F}{A} + \frac{2}{5V}$

So,

$$f_2 = f_1 \left(\frac{\frac{F}{A} + \frac{2}{5V}}{\frac{F}{A}} \right)^{5/7} = f_1 \left(1 + \frac{2A}{5VF} \right)^{5/7}, \quad V = A \cdot h = A \cdot \frac{v_s}{2f_1}$$

$$\Rightarrow \boxed{f_2 = f_1 \left(1 + \frac{4f_1}{5 v_s F} \right)^{5/7}}, \quad F = \underbrace{(2\ kg)}_{\text{mass of speaker}} \underbrace{(9.8\ m/s^2)}_{g}$$