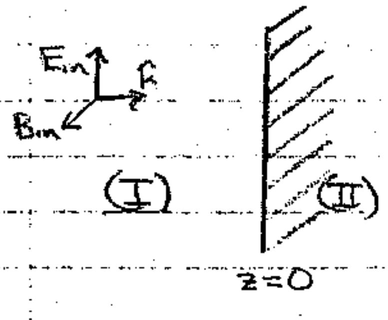
Fall 1999 #1

Reflection of light from a conductor

a). Suppose a linearly polarized monochromatic electromagnetic wave propagating in the z direction in a vacuum is incident on a perfect conductor at $z=0$. Assuming $E=0$ inside the conductor, calculate the reflected wave.



choose polarization of incoming wave along the x-axis

at interface boundary conditions are

(i) $(\vec{D}_2-\vec{D}_1)\cdot\hat{n} = 4\pi\sigma_f$ (iii) $(\vec{B}_2-\vec{B}_1)\cdot\hat{n}=0$

(ii) $(\vec{E}_2-\vec{E}_1)\times\hat{n}=0$ (iv) $\hat{n}\times(\vec{H}_2-\vec{H}_1)=\frac{4\pi}{c}l$

conditions (ii), (iii), and (iv) are automatically satisfied, and (i) becomes

$$E_{II}-E_{I}=0$$

now

$$E_I = (E_{in}e^{ikz}+E_{out}e^{-ikz})e^{-i\omega t}\,\hat{x}$$
$$B_I = (E_{in}e^{ikz}-E_{out}e^{-ikz})e^{-i\omega t}\,\hat{y}$$
$$E_{II}=0$$
$$B_{II}=0$$

so, at $z=0$ we get (ignoring time dependence)

$$E_{in}+E_{out}=0$$
$$E_{out}=-E_{in}$$

Thus, the reflected wave is

$$\boxed{\begin{aligned}\vec{E}_{ref} &= -E_{in}e^{-ikz-i\omega t}\,\hat{x}\\ \vec{B}_{ref} &= E_{in}e^{-ikz-i\omega t}\,\hat{y}\end{aligned}}$$

b) Now suppose the conductor is not perfect, but has high conductivity σ, with $\vec{J}=\sigma\vec{E}$. For what values of σ can you ignore the displacement current in Maxwell's equations in the conductor?

$$\nabla\times\vec{H} = \frac{4\pi}{c}\vec{J}_f + \underbrace{\frac{1}{c}\frac{\partial\vec{D}}{\partial t}}_{=\text{Displacement current}}$$

$$\nabla\times H = \frac{4\pi}{c}\sigma\vec{E} + \frac{\epsilon i\omega}{c}\vec{E}$$

in order to neglect displacement current, need:

$$4\pi\sigma \gg |\epsilon i\omega|$$

$$\sigma \gg \frac{\epsilon\omega}{4\pi}$$

$$\boxed{\sigma \gg \epsilon\omega}$$

c) In the regime where we can ignore the displacement current, calc the amplitude of the reflected wave.

(ii) $\to$ $E_{1\parallel} = E_{2\parallel}$ $\Rightarrow$ $E_{in} + E_{out} = E_T$ (v)

(iii) $\to$ $B_{1\perp} = B_{2\perp}$ $\Rightarrow$ $E_{in} - E_{out} = \underbrace{\frac{ck}{\omega}}_{=n \text{ (index of refraction)}} E_T$ (vi)

add (v) & (vi) to get $2E_{in} = E_T\left(1+\frac{ck}{\omega}\right)$

subtract (v)-(vi) to get $2E_{out} = E_T\left(1-\frac{ck}{\omega}\right)$

solving for E_{out} yields:

$$2E_{out} = \frac{(2E_{in})}{\left(1+\frac{ck}{\omega}\right)}\left(1-\frac{ck}{\omega}\right)$$

recall $\frac{k}{\omega} = v$

$$\boxed{\frac{E_{out}}{E_{in}} = \frac{\left(1-\frac{c}{v}\right)}{\left(1+\frac{c}{v}\right)}}$$

for $\sigma \to \infty$, $v \to 0$, so $E_{out} = -$

7.