e[illegible]on neutrino $|\nu_e\rangle$ and a muon neutrino $|\nu_\mu\rangle$ are possible neutrino states produced in an [illegible]periment. These states are linear combinations of eigenstates of the Hamiltonian:

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$$|\nu_e\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$$
$$|\nu_\mu\rangle = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle$$

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where:

$$H|\nu_1\rangle = \sqrt{p^2c^2 + m_1^2c^4}|\nu_1\rangle = E_1|\nu_1\rangle$$
$$H|\nu_2\rangle = \sqrt{p^2c^2 + m_2^2c^4}|\nu_2\rangle = E_2|\nu_2\rangle$$

Initially, a $|\nu_\mu\rangle$ was produced. Assuming neutrinos are moving at the speed of light, what is the probability of detecting a $|\nu_e\rangle$ after traveling a distance L?

We are given that $|\psi(0)\rangle = |\nu_\mu\rangle$, thus the time evolution of this state yields:

$$|\psi(t)\rangle = e^{\frac{-iHt}{\hbar}}|\psi(0)\rangle = -\sin\theta e^{\frac{-iE_1t}{\hbar}}|\nu_1\rangle + \cos\theta e^{\frac{-iE_2t}{\hbar}}|\nu_2\rangle$$

The probability of finding $|\nu_e\rangle$ is given by

$$P(t) = |\langle\nu_e|\psi(t)\rangle|^2$$

$$\langle\nu_e|\psi(t)\rangle = (\cos\theta\langle\nu_1| + \sin\theta\langle\nu_2|)\bullet\left(-\sin\theta e^{\frac{-iE_1t}{\hbar}}|\nu_1\rangle + \cos\theta e^{\frac{-iE_2t}{\hbar}}|\nu_2\rangle\right)$$

$$= \sin\theta\cos\theta\left(e^{\frac{-iE_2t}{\hbar}} - e^{\frac{-iE_1t}{\hbar}}\right)$$

$$P(t) = |\langle\nu_e|\psi(t)\rangle|^2 = \sin^2\theta\cos^2\theta\left(e^{\frac{-iE_2t}{\hbar}} - e^{\frac{-iE_1t}{\hbar}}\right)\left(e^{\frac{iE_2t}{\hbar}} - e^{\frac{iE_1t}{\hbar}}\right)$$

$$= \sin^2\theta\cos^2\theta\left(1 - e^{\frac{it(E_1-E_2)}{\hbar}} - e^{\frac{-it(E_1-E_2)}{\hbar}} + 1\right)$$

$$= \sin^2\theta\cos^2\theta\left(2 - 2\cos\left(\frac{t(E_1-E_2)}{\hbar}\right)\right)$$

Using the approximation c=L/t yields t=L/c. Furthermore,

$$E_1 - E_2 = pc\left(\sqrt{1 + \frac{m_1^2c^2}{p^2}} - \sqrt{1 + \frac{m_2^2c^2}{p^2}}\right)$$

can be simplified using the binomial expansion.

$$E_1 - E_2 \approx pc\left(1 + \frac{m_1^2c^2}{2p^2} - 1 - \frac{m_2^2c^2}{2p^2}\right) = \frac{c^3\Delta m^2}{2p^2}$$

Thus, the probability of finding an electron neutrino is:

$$P(t) = 2\sin^2\theta\cos^2\theta\left(1 - \cos\left(\frac{Lc^2\Delta m^2}{2\hbar p}\right)\right)$$