Fall 1999 #12

$V(r) = V_R(r) - iW_I(r)$ # of absorbing particles per unit Volume = N

$P(t) = \psi^*(t)\psi(t)$ (probality density of states)

ψ and ψ^* satisfy Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m}\nabla^2\psi + V(r)\psi \qquad -i\hbar\frac{\partial\psi^*}{\partial t} = \frac{-\hbar^2}{2m}\nabla^2\psi^* + V(r)\psi^*$$

$$\frac{\partial P}{\partial t} = \left(\frac{\partial\psi^*}{\partial t}\right)\psi + \psi^*\left(\frac{\partial\psi}{\partial t}\right)$$

$$i\hbar\frac{\partial P}{\partial t} = \frac{\hbar^2}{2m}(\nabla^2\psi^*)\psi - V^*(r)\psi^*\psi - \frac{\hbar^2}{2m}(\nabla^2\psi)\psi^* + V(r)\psi\psi^*$$

$$i\hbar\frac{\partial P}{\partial t} = \frac{\hbar^2}{2m}\left[(\nabla^2\psi^*)\psi - (\nabla^2\psi)\psi^*\right] + \left[-V_R - iW_I + V_R - iW_I\right]\psi\psi^*$$

$$= \frac{\hbar^2}{2m}\nabla\cdot(\psi\nabla\psi^* - \psi^*\nabla\psi) - 2iW\psi\psi^*$$

, where $\vec{J} \equiv (\psi\nabla\psi^* - \psi^*\nabla\psi)$ is the probability current density (see next page)

For one dimension travel

$$\psi(t) = e^{-iEt/\hbar}e^{ikx} \Rightarrow \psi\nabla\psi^* - \psi^*\nabla\psi = e^{-iEt/\hbar}e^{ikx}(-ik)e^{iEt/\hbar}e^{-ikx}$$

$$- e^{iEt/\hbar}e^{-ikx}(ik)e^{-iEt/\hbar}e^{ikx}$$

$$= (-ik) - ik = -2ik \Rightarrow \nabla\cdot(-2ik) = 0$$

Now

$$i\hbar\frac{\partial P}{\partial t} = -2iW\psi\psi^* = -2iWP(t)$$

$$P(t) = e^{-2Wt/\hbar}$$

But since it is the probability of states, aka the probability of the particle to be there

$$\Rightarrow P_{absorbed} = 1 - e^{-2Wt/\hbar}$$

Flux density $\propto$ $P(t)v$ since particle density $\propto$ $P(t)$

Flux density has units of #/Area · sec

$\frac{\#\text{ events}}{\text{sec}}$ = (Flux density)(cross section)

# of absorption events between time 0 and t is proportional to $P_{abs}(t)$.

$\Rightarrow$ After 1 sec

fraction of absorbed particles is $1-e^{-2W/\hbar}$

$$\left(1-e^{-2W/\hbar}\right) = \left(e^{-2W/\hbar} vN\right)(\sigma)$$

$$\boxed{\sigma = \left(\frac{e^{2W/\hbar}-1}{Nv}\right)}$$

$$\underbrace{\frac{\partial \vec{P}}{\partial t}}_{\text{time derivative of stuff}} + \underbrace{\nabla\cdot\vec{J}}_{\text{flow of stuff}} = \underbrace{-\frac{2}{\hbar}W\psi\psi^*}_{\text{source or sink of stuff}}$$

Now, $\vec{P}$ is the probability of particles that still exist and are not absorbed. So, $\vec{J}$ is the flow of the particles that still exist and are not absorbed. Obviously the # particles that exist and are not absorbed does not change on either side of region. This implies that the divergence of the $\vec{J}$ that corresponds to the flow of particles that still exist and are not absorbed is exactly zero ($\nabla\cdot\vec{J}=0$).

Thus,

$$\frac{\partial \vec{P}}{\partial t} = -\frac{2}{\hbar}W\psi\psi^*$$