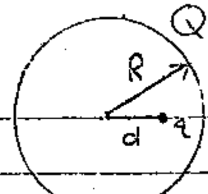
Fall 1999

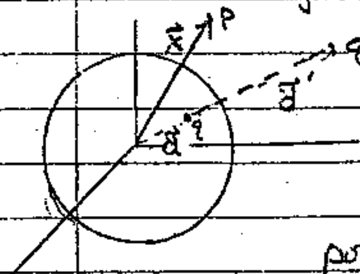
#3



A thin metallic shell of radius R carries a charge Q. A point charge q is introduced to the interior of the shell a distance d from its center. Find the force acting on this point charge.

(refer to Jackson sect 2.2, 2.3)

First consider charge inside a grounded, conducting sphere, with image charge(q') located outside the sphere.



The potential due to the two charges is

$$\Phi(\vec{x}) = \frac{q}{|\vec{x}-\vec{d}|} + \frac{q'}{|\vec{x}-\vec{d}'|} = \frac{q}{|x\hat{n}-d\hat{n}'|} + \frac{q'}{|x\hat{n}-d'\hat{n}'|}$$

potential must vanish at $|\vec{x}| = R$, thus

$$\Phi(r=R) = \frac{q}{R|\hat{n}-\frac{d}{R}\hat{n}'|} + \frac{q'}{d'|\hat{n}'-\frac{R}{d'}\hat{n}|}$$

thus, the image charge must satisfy

$$\frac{q}{R} = -\frac{q'}{d'} \qquad \frac{d}{R} = \frac{R}{d'}$$

$$\boxed{q' = -\frac{Rq}{d} \qquad d' = \frac{R^2}{d}}$$

The total charge induced on the surface of the sphere is $-q$ (Jackson pg 60). Now, if the sphere is isolated, any additional charge will distribute itself uniformly, so the force acting on the charge inside the sphere is the same as when the sphere is grounded, and given by:

$$F = \frac{qq'}{(d'-d)^2} = -\frac{q^2R}{d\left(\frac{R^2}{d}-d\right)^2} = -\frac{q^2R}{d}\left(\frac{d}{R^2-d^2}\right)^2$$

$$\boxed{F = -\frac{q^2Rd}{(R^2-d^2)^2}}$$