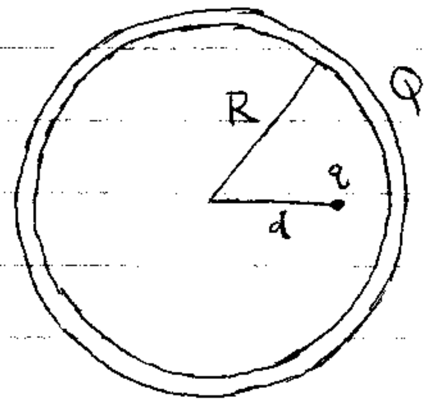
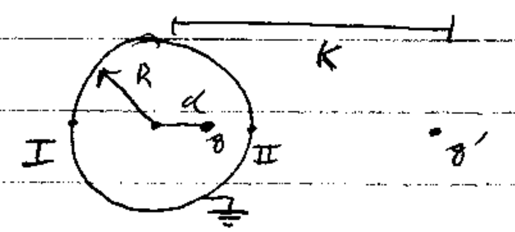
A thin metallic shell of radius R carries a charge Q. A point charge q is introduced into the interior of the shell a distance d from its center. Find the force acting on this point charge.



As there is no electric field due to Q inside of the sphere the only force on q is due to the image charge. So we can completely neglect Q and treat the sphere as uncharged (grounded even)



At point I & II the potential due to q and q' have to be zero.

(I) $\frac{q}{R+d} + \frac{q'}{K+R} = 0 \Rightarrow q' = -q\,\frac{(K+R)}{(R+d)}$ (1)

(II) $\frac{q}{R-d} + \frac{q'}{K-R} = 0 \Rightarrow q' = -q\,\frac{(K-R)}{(R-d)}$ (2)

sub. (1) into (2): $\frac{K+R}{R+d} = \frac{K-R}{R-d} \Rightarrow (K+R)(R-d) = (K-R)(R+d)$

$KR - Kd + R^2 - Rd = KR + Kd - R^2 - Rd$

so $Kd = R^2 \Rightarrow K = \frac{R^2}{d}$

hence $q' = -q\,\frac{\left(\frac{R^2}{d}+R\right)}{(R+d)} = \frac{-qR}{d}\,\frac{(R+d)}{(R+d)} = \frac{-qR}{d}$

So the force would be:

$$F = \frac{1}{4\pi\epsilon_0}\frac{-q q'}{(K-d)^2} = \frac{-q^2}{4\pi\epsilon_0}\frac{R}{d}\frac{1}{\left(\frac{R^2}{d}-d\right)^2} = \frac{-q^2}{4\pi\epsilon_0}\frac{Rd}{(R^2-d^2)^2}$$