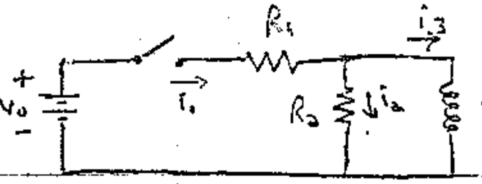
a)



Fall 1999 #5

$V_0 = 100\,V$
$R_1 = 10\,\Omega$ $\quad L = 10\,H$
$R_2 = 100\,\Omega$

(i) at $t=0$ switch is closed. Find total energy dissipated in R_2.
using the loop rule on the smaller loop yields

$$V_0 - i_1 R_1 - i_2 R_2 = 0 \qquad \Rightarrow \qquad i_1 = \frac{V_0 - i_2 R_2}{R_1}$$

for the larger loop, recall the voltage drop across an inductor is given by $V_L = L\frac{di}{dt}$
Thus the loop rule yields:

$$V_0 - i_1 R_1 - L\frac{d}{dt}\underbrace{(i_1 - i_2)}_{=i_3} = 0$$

$$\cancel{V_0} - \cancel{V_0} + i_2 R_2 - L\frac{d}{dt}\left(\frac{V_0}{R_1} - \frac{i_2 R_2}{R_1} - i_2\right) = 0$$

$$L\frac{d}{dt}\left(i_2\left(\frac{R_2}{R_1}+1\right)\right) = -i_2 R_2$$

$$L\left(\frac{R_2+R_1}{R_1}\right)\frac{di_2}{dt} = -i_2 R_2 \qquad \Rightarrow \frac{di_2}{dt} = \frac{-i_2 R_2 R_1}{L(R_2+R_1)}$$

$$\frac{di_2}{i_2} = -\frac{R_2 R_1}{L(R_2+R_1)}\,dt$$

$$\ln i_2 = \frac{-R_2 R_1 t}{L(R_2+R_1)} + \text{const} \qquad \Rightarrow i_2 = C\exp\left(\frac{-R_2 R_1 t}{L(R_2+R_1)}\right)$$

at $t=0$ all current flows through resistors, thus $i_2(0) = \frac{V_0}{(R_1+R_2)}$

$$i_2(t) = \left(\frac{V_0}{R_1+R_2}\right)\exp\left(\frac{-R_2 R_1 t}{L(R_2+R_1)}\right)$$

Energy dissipated given by $i^2 R$

$$W = \int_0^\infty i_2^2 R_2 = \frac{V_0^2 R_2}{(R_1+R_2)^2}\int_0^\infty e^{-2\alpha t}\,dt = \frac{V_0^2 R_2}{(R_1+R_2)^2}\left(-\frac{1}{2\alpha}\right)\underbrace{\left[e^{-2\alpha t}\right]_0^\infty}_{=\left[\frac{1}{e^\infty}-1\right]}$$

$$W = \frac{V_0^2 \cancel{R_2}}{2(R_2+R_1)^2}\,\frac{L(R_2+R_1)}{\cancel{R_2} R_1} = \boxed{\frac{L V_0^2}{2R_1(R_2+R_1)} = W}$$

ii) after opening switch, the loop rule gives $L\frac{di}{dt} + iR_2 = 0$

$$\frac{di}{dt} = -\frac{iR_2}{L} \Rightarrow i = Ce^{-\frac{R_2}{L}t}$$

at t_0, $i = V/R_1$, thus

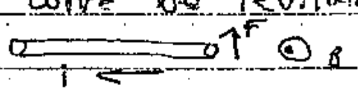
$$i(t) = \frac{V}{R_1} e^{-\frac{R_2 t}{L}}$$

the power dissipated is thus

$$W = \int_0^\infty \left(\frac{V_0}{R_1}\right)^2 R_2\, e^{-\frac{2R_2 t}{L}}\,dt = \frac{V_0^2}{R_1^2} R_2 \left(\frac{+L}{2R_2}\right)\left[e^{-\frac{2R_2 t}{L}}\right]_\infty^0$$

$$\boxed{W = \frac{V_0^2 L}{2R_1^2}}$$

b) Can a copper wire be levitated in Earth's magnetic field



$$F = i\,d\vec{L} \times \vec{B} \Rightarrow i\cancel{L}B = mg = \rho_m \cancel{L} A g$$

use $j = i/A$
↑ current density

$$\rho_m g = \frac{iB}{A} = jB$$

$$j = \frac{\rho_m g}{B} \Rightarrow i = \frac{\rho_m g A}{B}$$

Power $= i^2 R$

$$= \left(\frac{\rho_m g A}{B}\right)^2 \frac{\rho_e L}{A} \qquad R = \frac{\rho_e L}{A}$$

$$= \frac{\rho_m \rho_e g^2}{B}\underbrace{(AL)}_{=V}$$

$$\boxed{\frac{\text{Power}}{\text{Volume}} = \frac{\rho_m \rho_e g^2}{B}}$$