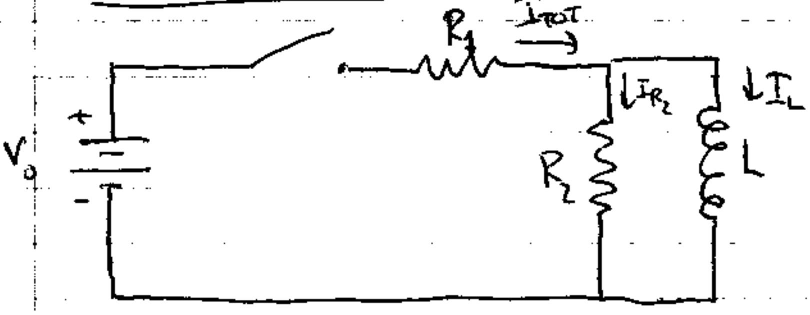
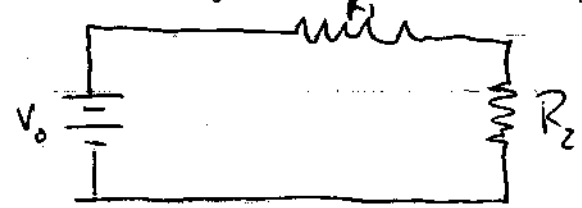
Fall '99 #5



Given:
$V_0 = 100V$,
$L = 10 H$,
$R_1 = 10\Omega$, $R_2 = 100\Omega$

& we know $V_L = -L\frac{dI}{dt}$, $V_R = IR$

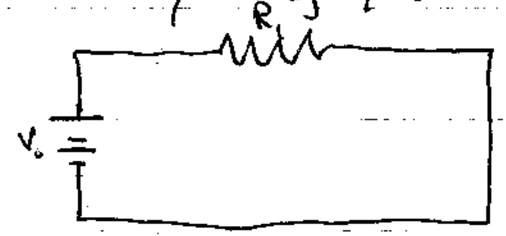
i.) • Initially (right after t=0), system like:



L resists ΔI

$\Rightarrow I_{R_2}(0) = \frac{V_0}{R_1+R_2}$

• Eventually though, after long time:



$\Rightarrow I_{R_2}(\infty) = 0$

• In general, for any electrical component $P = IV$, for resister here
$P = I_{R_2}^2 R_2 \Rightarrow$ find $I_{R_2}(t)$

• From Kirchoff's Laws; we can look @ sub-loops:

$V_0 - I_{TOT}R_1 - I_{R_2}R_2 = 0$ (#1)

$V_0 - I_{TOT}R_1 - L\frac{dI_L}{dt} = 0$ (#2)

#1 → #2:

$V_0 - V_0 + I_{R_2}R_2 - L\frac{dI_L}{dt} = 0$

$I_{R_2}R_2 - L\frac{d}{dt}(I_{TOT} - I_{R_2}) = 0$

$$\#1 \rightarrow I_{R_2} R_2 - L \frac{d}{dt}\left(\frac{V_0}{R_1} - I_{R_2}\frac{R_2}{R_1} - I_{R_2}\right) = 0$$

$$I_{R_2} R_2 + L\left(\frac{R_2}{R_1} + 1\right)\frac{d}{dt} I_{R_2} = 0$$

$$\rightarrow \frac{dI_{R_2}}{dt} = -I_{R_2}\left(\frac{R_2 R_1}{L(R_1+R_2)}\right)$$

- you can assume a form like

$$I_{R_2}(t) \sim e^{-\frac{R_2 R_1}{L(R_1+R_2)}}$$

- but we know IC $I(0) = \frac{V_0}{R_1+R_2}$, so

$$I_{R_2}(t) = \frac{V_0}{R_1+R_2} e^{-\frac{R_2 R_1}{L(R_1+R_2)}t}$$

- Now, $P = I_{R_2}(t)^2 R_2$, so

$$W = \int_0^\infty \left(\frac{V_0}{R_1+R_2}\right)^2 e^{-\frac{2R_2R_1}{L(R_1+R_2)}t} \cdot R_2 \, dt$$

$$= \frac{V_0^2 R_2}{(R_1+R_2)^2} \cdot \ominus \frac{L(R_2+R_1)}{2R_2R_1} e^{-\frac{2R_2R_1}{L(R_2+R_1)}t}\Big|_0^\infty$$

$$W = \frac{V_0^2 L}{2(R_2+R_1)} = 45.5\,\text{J}$$

ii) energy stored in L will all be dissipated by R_2 (assuming as usual negligible resistance within L

$$W_{R_2} = \frac{1}{2} L I_L^2(0) = \frac{1}{2} L\left(\frac{V_0}{R_1}\right)^2 = 500\,\text{J}$$

b.)

$$F_w = I\, d\vec{l} \times \vec{B}$$

$$F_g = mg$$

$$ILB = \rho_m V_a g$$

$$\rightarrow I = \frac{\rho_m g A}{B}$$

(know, you want in terms of I to get power)

$$P = I^2 R = \frac{\rho_m^2 g^2 A^2}{B^2} R$$

but $R = \frac{\rho_e L}{A}$, so

$$P = \frac{\rho_m^2 g^2 A L \rho_e}{B^2}$$

$$\Rightarrow \frac{P}{V} = \frac{\rho_m^2 g^2 \rho_e}{B^2}$$