QM S'99 #12

$$H_0 = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2}$$

a) $a\Psi_0 = 0$; $a = \sqrt{\frac{m\omega}{2\hbar}}\left(x + \frac{ip}{m\omega}\right)$

$$\Rightarrow \cancel{\sqrt{\frac{m\omega}{2\hbar}}}\left(x + \frac{ip}{m\omega}\right)\Psi_0 = 0 \Rightarrow x\Psi_0 + \frac{\hbar}{m\omega}\frac{d}{dx}\Psi_0 = 0$$

b) $\frac{\hbar}{m\omega}\frac{d\Psi_0}{dx} = -x\Psi_0 \Rightarrow \frac{d\Psi_0}{\Psi_0} = -\frac{m\omega}{\hbar}x\,dx$

$$\ln \Psi_0 = -\frac{m\omega}{2\hbar}x^2 + c \Rightarrow \Psi_0 = A_0 e^{-\frac{m\omega}{2\hbar}x^2}$$

c) $\Psi_1 = a^+\Psi_0$; $a^+ = \sqrt{\frac{m\omega}{2\hbar}}\left(x - \frac{ip}{m\omega}\right) = \sqrt{\frac{m\omega}{2\hbar}}\left(x - \frac{\hbar}{m\omega}\frac{d}{dx}\right)$

$$= \sqrt{\frac{m\omega}{2\hbar}}\left(x - \frac{\hbar}{m\omega}\frac{d}{dx}\right)\Psi_0 = \sqrt{\frac{m\omega}{2\hbar}}\left(x\Psi_0 - \frac{\cancel{\hbar}}{\cancel{m\omega}}\underbrace{\frac{d}{dx}e^{-\frac{m\omega}{2\hbar}x^2}}_{-\frac{m\omega}{\hbar}x\Psi_0}\right)$$

$$= \sqrt{\frac{m\omega}{2\hbar}}\,2A_0\,x\,e^{-\frac{m\omega}{2\hbar}x^2}$$

d) $U = e^{-ipb/\hbar}$; $b \in \mathbb{R}$ see Sakurai p. 48

This is just the finite translation operator with a translation of distance b.

So the ground state wave function is just given by:

$$\Psi_0' = A_0 e^{-\frac{m\omega}{2\hbar}|x-b|^2}$$