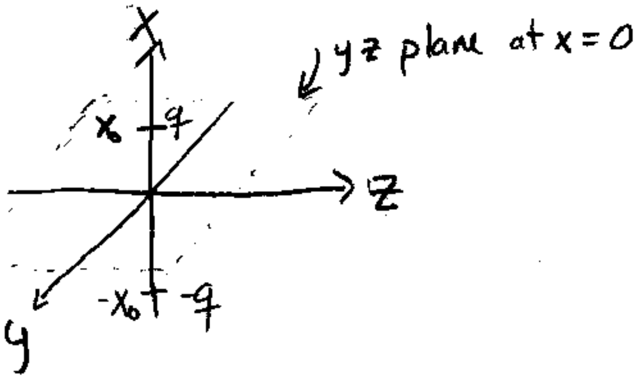
(a). consider a charge q that sits at rest, at a point $(x_0, 0, 0)$ near a conducting surface - the plane $x=0$. Calculate the force on the charge.

This is a method of images problem. See Griffiths p123 sec 3.2.3.

The charge q is attracted toward the plane, because of the negative induced charge.



So, the Force is (in cgs)

$$\boxed{\vec{F} = \frac{-q^2}{(2x_0)^2}\hat{x}}$$

(b) Calculate the time, T, for the charge to hit the conductor. Ignore any relativistic effects.

from part (a), we know that at $t=0$, the energy is

$$E_0 = -\frac{q^2}{4x_0}$$ ← this energy is half of what the energy would be if you had 2 point charges and no conductor (see Griffiths' p124)

once the charge starts moving the energy is given by

$$E = \frac{1}{2}m\dot{x}^2 - \frac{q^2}{4x}$$

solving for $\dot{x}$, we get

$$\dot{x} = \sqrt{\frac{2}{m}\left(E + \frac{q^2}{4x}\right)} \qquad (1)$$

Now, let's look at the definition of $\dot{x}$

$$\dot{x} = \frac{dx}{dt} \Rightarrow \int dt = \int \frac{dx}{\dot{x}}$$

using eq(1), we get

$$t - t_0 = \Delta t = T = \int_{x_0}^{0} \frac{dx}{\dot{x}} = \int_{x_0}^{0} \frac{dx}{\sqrt{\frac{2}{m}\left(\epsilon + \frac{q^2}{4x}\right)}}$$

This is actually as far as you can go w/out doing a numerical integration. Thus,

$$\boxed{T = -\int_{0}^{x_0} \frac{dx}{\sqrt{\frac{2}{m}\left(\epsilon + \frac{q^2}{4x}\right)}}}$$

(c) Now, consider the motion of a charge when there is also a magnetic field $\vec{B} = B_0 \hat{z}$. In the limit that B_0 is very large, calculate and sketch the motion of the charge.

The force is given by

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \quad , \quad \vec{v} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \dot{x} & \dot{y} & 0 \\ 0 & 0 & B_0 \end{vmatrix} = B_0(\dot{y}\hat{x} - \dot{x}\hat{y})$$

the velocity in the y direction is due to the magnetic field.

(i) case 1: $\vec{E}$ is constant in $B_0 \gg 1$ approx.

Here is the assumption: Assume that you only care about the trajectory of the charge far enough from the $x=0$ plane that the electric field is approximately constant with respect to the very large magnetic field B_0. That is, as $x \ll 1$, the Electric field will blow up. So, we need to avoid this region.

So, we have

$$\vec{F} = q\left[E_0 + B_0 \dot{y}\right]\hat{x} - q B_0 \dot{x} \hat{y}$$

Now, the equations of motion are

$$\ddot{x} = \frac{qE_0}{m} + \frac{qB_0}{m}\dot{y} = \omega\left[\frac{E_0}{B_0} + \dot{y}\right]$$

$$\ddot{y} = -\frac{qB_0}{m}\dot{x} = -\omega\dot{x}$$

where ω is derived from

$$\frac{mv^2}{R} = qvB \quad \& \quad v = \omega R$$

The coupled differential equations from the previous page are solved in Griffiths' p 206. The solutions are

$$x(t) = c_2 \cos(\omega t) - c_1 \sin(\omega t) + c_4$$
$$y(t) = c_1 \cos(\omega t) + c_2 \sin(\omega t) - \frac{E_0}{B_0}t + c_3$$

Applying initial conditions, we get

$$\dot{x}(t=0) = 0 = -c_1\omega \Rightarrow c_1 = 0$$
$$x(t=0) = X_0 = c_2 + c_4 \Rightarrow c_4 = X_0 - \frac{E_0}{\omega B_0}$$
$$\dot{y}(t=0) = 0 = c_2\omega - \frac{E_0}{B_0} \Rightarrow c_2 = \frac{E_0}{\omega B_0}$$
$$y(t=0) = 0 = \cancel{c_1}^{0} + c_3 \Rightarrow c_3 = 0$$

So, we have

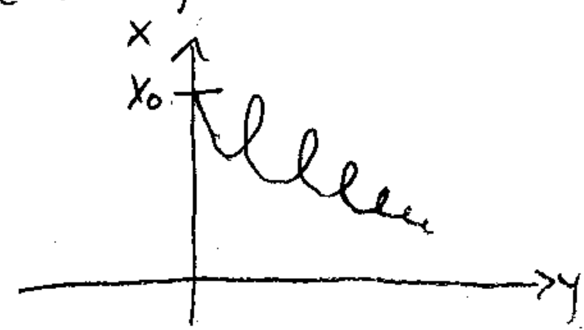
This cannot be correct. There is no acceleration of the charge toward $x=0$.

$$\boxed{\begin{aligned} x(t) &= \frac{E_0}{\omega B_0}\cos(\omega t) + X_0 - \frac{E_0}{\omega B_0} = \frac{E_0}{\omega B_0}\left[\cos(\omega t) - 1\right] + X_0 \\ y(t) &= \frac{E_0}{\omega B_0}\left[\sin(\omega t) - \omega t\right] \end{aligned}}$$

note: $\frac{E_0}{\omega B_0} = \frac{mE_0}{qB_0^2} \simeq 0$ when B_0 is very large

This yields the following equations of motion: $x(t) = X_0$, $y(t) = 0$

But, the charge slowly fall to the $x=0$ plane A-like so



So Case 1 seems to have failed!

(ii) case 2 : $\bar{E}$ is not constant in $B_0 >> 1$ approx.

here we have

$$\ddot{x} = \omega \left[\frac{1}{B_0}\frac{(-q)}{(2x)^2} + \dot{y}\right] \Rightarrow \dot{x} = \omega\left[\frac{+q}{4B_0 x} + y\right] + c_1$$

$$\ddot{y} = -\omega \dot{x} \Rightarrow \dot{y} = -\omega x + c_2$$

substituting $\dot{y}$ into our expression for $\ddot{x}$ yields

$$\ddot{x} = \omega\left[\frac{-q}{4B_0 x^2} - x + \frac{c_2}{\omega}\right]$$

$\Rightarrow$ Mathematica does not know how to solve this $\ddot{\frown}$