

The Life and Times of Emmy Noether

Contributions of Emmy Noether to Particle Physics*

Nina Byers

Physics Department, UCLA, Los Angeles, CA 90024

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Abstract

The contributions of Emmy Noether to particle physics fall into two categories. One is given under the rubric of Noether's theorem, and the other may be described as her important contributions to modern mathematics. In physics literature, the terminology Noether's theorem is used to refer to one or another of two theorems, or their converses, proved by Noether. These will be discussed along with an historical account of how they were discovered and what their impact has been. This paper also gives, for physicists, an overview of the important role of Emmy Noether's work in the development of modern mathematics. In addition a brief biography is given.

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DOROTHY HODGKIN 1910-1994

I would like to dedicate this lecture to Dorothy Crowfoot Hodgkin, the chemist and x-ray crystallographer, who died last Friday. She was one of the great scientists of our century. Using x-ray crystallographic methods she got out the chemical and three-dimensional structure of complicated molecules, changing organic chemistry forever and breaking ground for modern biology. She was awarded the Nobel Prize in 1964 for her work, particularly for getting out the structure of penicillin and vitamin B_{12} . But this was only a step along the way to the pinnacle of her scientific career. In 1969 she got out the structure of insulin, thirty-five years after she began work on this molecule. The importance of these discoveries for chemistry, modern biology and medicine cannot be overstated.

I. INTRODUCTION

Dorothy Hodgkin, Emmy Noether, and an eminent physicist here with us today, Chien Shiung Wu, are examples of women whose love of science enabled them to make great contributions in the face of daunting adversity. In the nineteenth century women were not admitted into universities and laboratories. In Germany and Austria the formal education of women ended at age fourteen. In 1898 the Academic Senate of the University of Erlangen, where Emmy Noether's father was professor, declared the admission of women students would "overthrow all academic order." [1] Women with intellectual interests, born toward the end of the 19th century, worked as governesses and language teachers from age fourteen until the universities finally admitted women. Some of these became great scientists - Marie Curie, Lise Meitner, Emmy Noether. As the 19th century drew to a close, the exclusion of women from academic and intellectual life began to be breached. Then and in the first few decades of the 20th century, womens' colleges were founded, women were admitted to universities and the process whereby it became possible for women to participate in scientific discovery began.

In 1900 Emmy Noether was eighteen, and women were finally permitted to attend lectures at the University in Erlangen. They were not allowed to matriculate, only to attend lectures - with permission of the instructor. Some lecturers refused to lecture if there was a woman in the room. Emmy Noether was thus at the forefront of the entry of women into academic life. One might speculate whether it is a remarkable accident that a woman of genius was among the first, or whether the social, psychological and emotional barriers against women doing science were so formidable that only women of extremely high ability and determination were able to overcome them.

Emmy Noether was one of the great mathematicians of the 20th century, as all mathematicians will attest. Not only did she discover the oft quoted theorem which relates symmetries and conservation laws, she contributed original and fundamental ideas to modern mathematics. The importance of modern mathematical ideas and tools to discoveries in theoretical elementary particle physics this century is self-evident.

It is fitting therefore that we acknowledge her contributions at this Conference on the History of Original Ideas and Basic Discoveries in Particle Physics. To discuss her contributions to particle physics, it is useful to separate them into two groups. One is centered around the theorem we call Noether's theorem, and the other her seminal contributions to the development of modern mathematics which has been so influential in theoretical particle physics. I will discuss the theorem in the next section, its importance, and the historical context in which it was discovered. The theorem was published in 1918 and essentially ignored in physics literature for forty years. There is something of a puzzle as to why it lay fallow for so long since its relevance to physics, and in particular quantum mechanics, is so clear to us today. From our modern perspective the theorem reduces the search for conservation laws and selection rules to the systematic study of the symmetries of the Lagrangian, and conversely also leads from observed conservation laws to the discovery of symmetries. In section III is some history which may be relevant to why Noether's theorem was so rarely quoted in physics literature from 1918 to 1958. Section IV is a overview for physicists of her original and highly influential contributions to modern mathematics, in particular abstract

algebra. Section V is a brief biographical sketch of her life and work including some details about her father and brother who were also mathematicians. A list of her published papers is in Appendix A; in Appendix B is her summary of the work published before 1919; in Appendix C are 72 articles in physics and mathematics journals listed in a recent issue of Current Contents whose titles refer to Noether charges, Noether currents, Noether theorem, etc. - of these 26 appear to be in journals of pure mathematics and the rest in physics and mathematical physics journals!

II. THE THEOREM

The theorem we so often quote was published in a paper entitled *Invariante Variationsprobleme* in the Göttingen Nachrichten in 1918. [2] It is a very important paper for physics because it proves very generally the fundamental relation of symmetries and conservation laws. The theorem reduces the search for conservation laws and selection rules to a systematic study of the symmetries of the system, and vice versa, for systems governed by an action principle whose action integral is invariant under a continuous group of symmetry transformations. Noether's paper combines the theory of Lie groups with the calculus of variations, and proves two theorems, referred to as I and II, and their converses. Both theorems and their converses are called Noether's theorem in the physics literature. Theorem I pertains to symmetries described by finite-dimensional Lie groups such as the rotation group, the Lorentz group, SU(3) or U(1). Theorem II applies for infinite-dimensional Lie groups such as gauged U(1) or SU(3) or the group of diffeomorphisms of general relativity. It is likely that theorem II was of principal interest at the time the paper was written because, applied to the theory of general relativity, from it one obtains energy-momentum conservation as a consequence of the general coordinate invariance of the theory. Similarly, and somewhat more simply, one may obtain current conservation as a consequence of gauge invariance in electrodynamics. Emmy Noether did this work soon after Einstein completed the theory of relativity and Hilbert derived the field equations from an action principle. Hilbert was con-

cerned by the apparent failure of ‘proper’ energy conservation laws in the general theory. [3] It is characteristic of Emmy Noether that, having begun this work in response to Hilbert’s questions regarding energy-momentum conservation in the general theory, she got results of utmost generality and found theorems that not only illuminate this question but many other conservation laws as well. I will describe her two theorems and the historical setting in which they became known.

Theorem I applies when the symmetry group is a finite-dimensional Lie group; a Lie group with a finite number N of infinitesimal generators. Examples are the Lorentz group with $N = 6$, and ungauged $SU(3)$ and $U(1)$ with $N = 8$ and $N = 1$, respectively. These generators are the elements of the Lie algebra. Theorem I states, if the system is invariant with respect to the Lie group, there is a conserved quantity corresponding to each element of the Lie algebra. The result is very general and holds for discrete and continuous, classical and quantum systems. For a field theory, theorem I states that there is a locally conserved current for each element of the algebra; i.e., that there are N linearly independent currents $j_\mu^{(a)}(x)$ which obey $\partial^\mu j_\mu^{(a)} = 0$ where a is a Lie algebra label and $\mu = 0, 1, 2, 3$ a space-time label. Thus one has

$$\partial^\mu j_\mu^{(a)} = \partial_0 j_0^{(a)} + \nabla \cdot \mathbf{j}^{(a)} = 0 \quad (1)$$

for each infinitesimal generator of the group. From this we obtain conservation of the corresponding charge

$$Q^{(a)} = \int d^3x j_0^{(a)}. \quad (2)$$

For quantum systems these charges are operators whose commutation relations are those of the Lie algebra.

Theorem II applies when the symmetry group is an infinite-dimensional Lie group (not the limiting case of $N \rightarrow \infty$ for which theorem I continues to apply). Examples are the gauged $SU(3)$ and $U(1)$ groups of QCD and QED, and the group of general coordinate transformations of general relativity. Theorem II states that certain dependencies hold

between the left hand sides of the Euler equations of motion when the action is invariant with respect to an infinite-dimensional Lie group. In the case of general relativity, using Hilbert's Lagrangian and the invariance of the action under general coordinate transformations, the dependencies of theorem II are Bianchi identities. The four Bianchi identities $G_{\mu\nu;\mu} = 0$ give the energy-momentum conservation law, as can be seen from the following. For Einstein gravity coupled to electromagnetism and/or matter, the field equations are $G_{\mu\nu} = -8\pi\kappa T_{\mu\nu}$ where $T_{\mu\nu}$ is the energy-momentum tensor of the electromagnetic and/or matter fields. Theorem II thus gives $T_{\mu\nu;\mu} = 0$, which is the law of energy-momentum conservation in the general theory. Similarly, for QED theorem II gives current conservation as a consequence of gauge invariance.

For the mixed case where the symmetry group is the union of a finite-dimensional and an infinite-dimensional Lie group, Noether found both types of results; i.e., conservation laws and dependencies.

The paper was submitted to the University of Göttingen in 1919 as her Habilitation thesis. Actually Hilbert had tried to obtain a university Habilitation for Noether in 1915 when she came to Göttingen. Consideration was refused by the academic senate on grounds she was a woman, and Hilbert uttered his famous quote "I don't see why the sex of the candidate is relevant - this is after all an academic institution not a bath house." The Habilitation was granted in 1919. It is interesting to read how she describes her results in her submission. She says the paper "deals with arbitrary finite- or infinite-dimensional continuous groups, in the sense of Lie, and discloses the consequences when a variational problem is invariant under such a group. The general results contain, as special cases, the theorems on first integrals as they are known in mechanics and, furthermore, the conservation theorems and the dependencies among the field equations in the theory of relativity - while on the other hand, the converse of these theorems is also given ..." [4] In the Abstract to the paper she wrote "The variational problems here considered are such as to admit a continuous group (in Lie's sense); the conclusions that emerge for the differential equations find their most general expression in the theorems formulated in section I and proved in the following

sections. Concerning differential equations that arise from problems of variation, far more precise statements can be made than about arbitrary differential equations admitting a group, which are the subject of Lie's researches. For special groups and variational problems, this combination of methods is not new; I may cite Hamel and Herglotz for special finite-dimensional groups, Lorentz and his pupils (e.g., Fokker, Weyl and Klein) for special infinite-dimensional groups. In particular Klein's second Note and the present developments have been mutually influenced by each other. In this regard I refer to the concluding remarks of Klein's Note." [5]

The Note of Klein she refers to is entitled *Über die Differentialgesetze für die Erhaltung von Impuls and Energie in der Einsteinschen Gravitationstheorie*. It ends with an acknowledgement to Noether saying "I must not fail to thank Fr. Noether again for her valuable participation in my new work... Her general treatment is given in these Nachrichten in a following Note." His work was presented to the Gesellschaft der Wissenschaften at a 19 July 1918 meeting, and he says he presented her more general results the following week. [6]

Noether's interest in the general theory was somewhat aside from the main path of her mathematical research as reflected in her publication list (Appendix A) but very understandable since she came to work in Göttingen in 1915 at Hilbert's invitation, and Hilbert says he asked her to look into the question of energy conservation in Einstein's theory. [3] Göttingen, at that time, was the world center of mathematics; Hilbert had assembled there a stellar array of mathematicians. Felix Klein, Hermann Minkowski, and Karl Schwarzschild were among them. There was intense interest in the general theory of relativity. Hermann Weyl said "Hilbert was over head and ears in the general theory, and for Klein the theory and its connection with his old ideas of the Erlangen Program brought the last flareup of his mathematical interests and production." [7] Noether published two papers directly relating to the general theory - No. 12 and 13 in the her publication list. Weyl characterized these papers as follows. "For two of the most significant aspects of general relativity theory she gave the correct and universal mathematical formulation: first, the reduction of the problem of differential invariants to a purely algebraic one by the use of 'normal coordinates'; and

second, identities between the left sides of Euler's equations of a variational problem which occur when the [action] integral is invariant with respect to a group of transformations involving arbitrary functions." The paper we are considering here is the second of these two. It is interesting to examine further the historical situation. In the summer of 1915 Einstein gave six lectures in Göttingen on generalizing the special theory of relativity to include gravity. At this time, according to Pais [8], he did not yet have the theory completed but he felt he had '... succeeded to convince Hilbert and Klein'. In the fall, Einstein found, at last, found the correct field equations. At the same time Hilbert also got the same equations by writing a Lagrangian for the theory and deriving the field equations from an action principle. Weyl was very impressed, as was everyone, and he quickly wrote his book *Raum - Zeit - Materie*. The first edition was published in 1918. It begins " Einstein's theory of relativity has advanced our ideas of the structure of the cosmos a step further. It is as if a wall which separated us from truth has collapsed." [9]

Hilbert wrote an article entitled *Grundlagen der Physik* and remarked there on the failure in the general theory of ordinary laws of energy- momentum conservation; Klein published a correspondence with Hilbert on this. [3] Proof of local energy conservation is not clear as it is in Newtonian theories. The conservation laws in those theories were called "proper" by Hilbert and he found that they failed in the general theory. After proving that the by now accepted form of the energy-momentum conservation law follows from the invariance of the theory under general coordinate transformations, Noether concludes her paper with a section entitled A HILBERTIAN ASSERTION that begins " From the foregoing, finally, we also obtain the proof of a Hilbertian assertion about the connection of the failure of proper laws of conservation of energy with general relativity, and prove this assertion in a generalized group theory version." She proved that generally one has what they then called improper energy relationships when the symmetry group is an infinite-dimensional Lie group, and in addition to the general theory of relativity she gave another example of this. It was her style, starting from a specific case, to get the most general results.

The generality of her results is characteristic of the whole body of her work. The overall

distinguishing characteristic of her major contributions to modern mathematics stems from her ability to abstract matters of general importance from details. According to her student van der Waerden, her work was guided by a maxim he described as follows. [10] “Any relationships between numbers, functions and operations only become transparent, generally applicable, and fully productive after they have been isolated from their particular objects and been formulated as universally valid concepts. Her originality lay in the fundamental structure of her creative mind, in the mode of her thinking and in the aim of her endeavors. Her aim was directed specifically towards scientific insight.”

In the 1916 - 1918 period her work was widely recognized. Einstein wrote to Hilbert in the spring of 1918 “ Yesterday I received from Miss Noether a very interesting paper on invariant forms. I am impressed that one can comprehend these matters from so general a viewpoint. It would not have done the Old Guard at Göttingen any harm had they picked up a thing or two from her. She certainly knows what she is doing.” [1] He is probably referring to No. 12 in Appendix A. This is the only one of Noether’s papers cited in in Pauli’s 1921 *Encyklopaedie der mathematischen Wissenschaften* article on relativity. It seems odd that her didn’t also reference No. 13. Perhaps this was a harbinger of things to come. In the twenties and thirties, and indeed for about forty years, Noether was rarely cited in the literature though her results were given often. It is not clear why this is so. Perhaps it is because of an ambiguity having to do with Klein’s Note. In the acknowledgement he makes to her contributions to his work, there is perhaps some insinuation that he was somehow responsible for her results. [6]

There is no paucity of references to Noether’s theorems in contemporary literature. As regards theorem II, Peter G. Bergmann wrote in 1968: “Noether’s theorem forms one of the cornerstones of work in general relativity. General relativity is characterized by the principle of general covariance according to which the laws of nature are invariant with respect to arbitrary curvilinear coordinate transformations that satisfy minimal conditions of continuity and differentiability. A discussion of the consequences in terms of Noether’s theorem would have to include all of the work on ponderomotive laws, ... ” [1] Feza Gursey

wrote in 1983: “The key to the relation of symmetry laws to conservation laws is Emmy Noether’s celebrated theorem. ... Before Noether’s theorem the principle of conservation of energy was shrouded in mystery, leading to the obscure physical systems of Mach and Ostwald. Noether’s simple and profound mathematical formulation did much to demystify physics. ... Since all the laws of fundamental physics can be expressed in terms of quantum fields which are associated with symmetry groups at each point and satisfy differential equations derived from an action principle, the conservation laws of physics and the algebra of time-dependent charges can all be constructed using Noether’s methods. The only additional conserved quantities not connected with the Lie algebra are topological invariants that are related to the global properties of the fields. These have also become important in the last few years. With this exception, Noether’s work is of paramount importance to physics and the interpretation of fundamental laws in terms of group theory.”

Now Noether’s theorem is a basic tool in the arsenal of the theorist, and is taught in every class on quantum field theory and particle physics. It is curious that it seems to have lain fallow in the physics literature for nearly forty years being mentioned very rarely from 1920 to 1960. In the next section are some further comments and conjectures regarding this.

III. A PUZZLE

The puzzle is why were there so few references to E. Noether in the physics literature for nearly forty years? [11] Now her name appears very frequently, and most textbooks on classical and quantum mechanics and classical and quantum field theory have sections entitled Noether’s theorem. Actually her results did not fall into obscurity but they were often given without a reference to her. This may have begun with Hermann Weyl’s important book *Raum - Zeit - Materie* in which he derives the energy-momentum conservation law for relativity from general coordinate invariance. He does not refer to Klein or Noether in the text. In a footnote he references the Klein paper, and adds “Cf., in the same periodical, the general formulations given by E. Noether.” The English version in which one finds this is a

translation of a 1919 edition. [9] In the first edition, dated Easter 1918 in Mecklenburg, he gets an energy-momentum conservation law from the field equations. Obviously he was not aware then of Noether's theorem and Klein's Note. In the preface to the 1919 edition he says "Chapter IV, which is in the main devoted to Einstein's theory of gravitation, has been subjected to a very considerable revision in consideration of the various important works that have appeared, in particular those that refer to the Principle of Energy-momentum." Perhaps because Weyl's book was very important, and he did not mention Noether's theorem, others followed suit.

There is one important book written in the twenties that mentions Noether's theorem. A short subsection devoted to Noether's theorem is in Courant and Hilbert's *Methods of Mathematical Physics*; the German edition was first published in 1924. [12]

Perhaps a more substantial reason for the paucity of references to Noether's theorem in the twenties and thirties, than that Weyl didn't mention it, is that her theorems were not felt to be of fundamental importance. In that period, energy conservation and general relativity were not as firmly established as they are now. [13] Of course no one doubted macroscopic energy conservation; the first law of thermodynamics had been firmly established by 1850. But the discovery of radioactivity, particularly the continuous β spectrum, raised serious doubts regarding energy conservation as a fundamental principle. Though Chadwick had presented evidence of a continuous β spectrum in 1914, his results were not definitive and some thought that the electrons were monoenergetic and the observed continuous β spectrum an experimental artifact. Lise Meitner was among those who believed that energy conservation was a fundamental principle and that there must be narrow lines underlying the β spectrum. It was only in 1927 that Chadwick and Ellis gave convincing evidence in the form of calorimetric measurements that the β spectrum was continuous. Meitner then confirmed those results in her own laboratory, and this provoked Pauli's proposal of the neutrino in December 1929. [14] Though energy-momentum conservation had been clearly demonstrated experimentally in Compton scattering in 1925, Pauli's neutrino hypothesis did not immediately reinstate energy conservation as a fundamental principle. For example,

Bohr proposed energy nonconservation in nuclear processes in his Faraday lecture at Caltech in 1930. He wrote to Mott in October 1929 “I am preparing an account on statistics and conservation in quantum mechanics in which I also hope to give convincing arguments for the view that the problem of β -ray expulsion lies outside the reach of the classical conservation principles of energy and momentum.” [13] Pais writes that Bohr continued to consider the possibility that energy is not conserved in β -decay until 1936. You might think that Fermi’s incorporation of Pauli’s neutrino hypothesis in his theory of beta decay (published December 1933) would have reestablished the credibility of energy conservation as a fundamental principle. However, in 1936 there were experimental indications (later proved false) of failure of the conservation laws in the Compton effect and, for example, Dirac wrote a paper entitled ‘Does conservation of energy hold in atomic processes?’ [15] It was not until 1939 that measurements of β -spectra in allowed transitions confirmed Fermi’s theory. [16] Energy conservation in atomic processes was not in doubt, at least not for long; but it does seem that energy conservation as a fundamental principle was in doubt. Perhaps at some level it remained so all the way until 1956, when the definitive experimental verification of energy conservation in β decay was achieved with the direct detection of ν_e by Reines and Cowan. [17]

With the advent of quantum mechanics, one might have thought that Noether’s theorem would have been invoked. The connection between symmetries and conservation laws was of fundamental interest. Nevertheless it is remarkable that the only reference to Emmy Noether in Weyl’s *Theory of Groups and Quantum Mechanics* is to her paper generalizing the Jordan-Hölder theorem. [18] He uses her theorem II in his treatment of the Dirac electron in interaction with the electromagnetic field, but without reference to her paper. From gauge invariance of the action, he obtains conservation of current and then shortly thereafter says “Just as the theorem of conservation of electricity follows from the gauge invariance, the theorems for conservation of energy and momentum follow from the circumstance that the action integral, formulated as in the general theory of relativity, is invariant under arbitrary (infinitesimal) transformations of coordinates.” Perhaps he omitted referencing

her 1918 paper because by the time his book on group theory and quantum mechanics was written (1928), her more recent work overshadowed, for mathematicians, her theorems on symmetries and conservation laws. Nevertheless, she might have benefited from multiple citations of her work. Her status in the University was far below what she merited on the basis of her accomplishments and ability. Weyl had been a visitor to Göttingen in 1926-27. In the address he gave at her memorial service he said “When I was called permanently to Göttingen in 1930, I earnestly tried to obtain a better position for her, because I was ashamed to occupy such a preferred position beside her whom I knew to be my superior as a mathematician in many respects. I did not succeed... Tradition, prejudice, external considerations weighted the balance against her scientific merits and scientific greatness, by that time denied by no one. In my Göttingen years, 1930-1933, she was without doubt the strongest center of mathematical activity there, considering both the fertility of her scientific research program and her influence upon a large circle of pupils.” [7]

Perhaps another reason Noether’s theorem was not given much publicity was because it may have felt awkward for pre-WWII authors to have credited a woman for an important contribution to their work.

From a contemporary perspective it seems surprising that Weyl did not use Noether’s theorem I to obtain, for example, conservation of angular momentum from rotational invariance. This, however, doesn’t fit into the approach of his book because he uses a Hamiltonian rather than a Lagrangian formulation of quantum mechanics.

In the 1950’s when Lagrangian formulations became more prevalent, references to Noether’s theorem began appear in the literature. Kastrup describes the major papers that seem to bring it forward. [11] The first quantum field theory text I have found that mentions Noether’s theorem is Bogoliubov and Shirkov’s *Introduction to the Theory of Quantized Fields*. [19] This book presents classical and quantum field theories from a Lagrangian point of view, and devotes a subsection to Noether’s theorem (theorem I) in what is essentially the first chapter. Gregor Wentzel’s book *Quantum Theory of Fields*, widely used in the forties and fifties, does not use it or refer to it, though in a footnote to the section entitled

Conservation Laws he remarks that “the validity of the conservation laws is known to be connected with certain invariance properties of the Hamiltonian.” [20] In the text he derives energy-momentum conservation when the Hamiltonian does not depend upon space-time coordinates by construction of a divergence-free symmetric energy-momentum tensor using the field equations. His book generally gives a Hamiltonian rather than a Lagrangian formulation of quantum field theory. In the footnote mentioned above, Wentzel references Pauli and Heisenberg [21]. In their famous papers on quantum field theory, there is no reference to Noether. There is also a Hamiltonian approach to the subject.

The frequency with which Noether’s theorem is referred to in physics literature, particularly particle physics literature, increased substantially after 1958. This was the year that the Feynman and Gell-Mann paper on the V-A theory of weak interactions was published. [22] Though no reference is made to Noether’s theorem, Feynman and Gell-Mann clearly point to the connection of conserved currents and symmetries. They propose in that paper the conserved vector current (CVC) hypothesis, observing that the decay rates of the muon and O^{14} give nearly equal values for the Fermi coupling constant. From this observation they suggest that the Fermi coupling constant may be a weak charge related to the conserved weak vector current as in (2). Like electric charge, it appears that it is not renormalized by the strong interactions and is the same for leptons and hadrons. Probably with reference to conservation of the electromagnetic current as a consequence of gauge invariance, they presciently seem to be suggesting that another gauge principle may be involved; the final sentence of their paper reads in part: “it may be fruitful to analyze further the idea that the vector part of the weak coupling is not renormalized; ... and to study the meaning of the transformation groups which are involved.” Another paper that was influential at around the same time was Schwinger’s 1957 *Annals of Physics* paper *A Theory of the Fundamental Interactions*. [23] In his theory, internal symmetries are described by finite-dimensional Lie groups and he uses, without reference to Noether, her theorem I. Indeed it plays an important role in his theory.

It seems to me that these papers along with the coming back into vogue of Lagrangian

field theory, led people to feel that Noether's theorem was important or, anyway, useful. Previously, and to some extent still at that time, people used a Hamiltonian approach for theoretical elementary particle physics even though Schwinger's formulation of quantum field theory in terms of an action principle had been enormously influential. Up until this time and even a bit beyond, most theorists were not thinking of theories of strong or weak interactions as Lagrangian field theories governed by an action principle. [24] Schwinger's 1957 paper is somewhat exceptional in this respect and perhaps, for some at least, led the way. As long as Lagrangian field theory was not seen as the starting point for a theory of elementary particles, Noether's theorem was not as consequential as it later became. Later when theorists began to use path integrals, Lie groups, and gauge symmetries, Noether's theorem became a basic tool in their arsenal.

It may be an amusing coincidence that two of the possible roadblocks to frequent mention of Noether's theorem in the older literature disappeared at about the same time. Final confirmation of the principle of energy conservation by Reines and Cowan's direct detection of ν_e occurred at roughly the same time as widespread recognition of the importance of Lie groups in Lagrangian formulations of quantum field theories began.

Perhaps we will learn that energy-momentum conservation is not a fundamental principle after all; i.e., that the diffeomorphism symmetry of space-time is violated at small distances. Nevertheless Noether's theorem will remain an important contribution to physics because it gives, in general, the relation between conservation laws and symmetries. Furthermore the theorem formulated by Noether with such depth and generality has contributed very importantly to modern physics both in the discoveries of symmetries of fundamental interactions and in finding the dynamical consequences of symmetries. I believe I would not be alone in asserting that her theorems have played a key role in the development of theoretical physics.

IV. CONTRIBUTIONS BEYOND THE THEOREM

As important as the theorem is, it by no means sums up her contributions to modern physics. From her point of view, and that of her mathematical colleagues, the two 1918 papers constituted a tangent to a main road of accomplishment. This road was to establish modern abstract algebra. It is self-evident that modern mathematics is, and has been, a very important contributor to discovery in particle physics. Modern abstract algebra profoundly affected modern mathematics in general; to quote Michael Atiyah “Modern mathematics, in all its branches, has been influenced by a more liberal and ambitious use of algebra. In recent years this is also increasingly true of theoretical physics. Lie groups, commutation relations, supersymmetry, cohomology, and representation theory are widely used in theoretical models for particle physics. Emmy Noether’s belief in the power of abstract algebra has been amply justified.” [25] Nathan Jacobson wrote in the introduction to her collected works that “Emmy Noether was one of the most influential mathematicians of this century. The development of abstract algebra, which is one of the most distinctive innovations of twentieth century mathematics, is largely due to her - in published papers, in lectures, and in personal influence on her contemporaries.” [26] Concepts, methods and results in group theory, algebraic topology, cohomology theory, homotopy theory, etc. are valuable tools for understanding physics. To give some recent examples, methods and concepts from algebraic topology are very usefully employed in analytic studies of gauge field theories on the lattice [27]; and higher homotopy groups are found useful in analyzing possible forms of spontaneous symmetry breaking. [28]

In this section I will give a brief overall summary of Emmy Noether’s contributions drawn principally from writings of Weyl [7], Jacobson [26] and van der Waerden [10]. Since we are not mathematicians, it is difficult to give here a complete and accurate account of her major contributions. A list of her published papers is given in Appendix A. Hermann Weyl said, however, that “one cannot read the scope of her accomplishments from individual results of her papers alone; she originated above all a new and epoch-making style of thinking

in algebra.” He writes about her work as follows. “Emmy Noether’s scientific production fell into three clearly distinct epochs; (1) the period of relative dependence, 1907-1919; (2) the investigations grouped around the general theory of ideals 1920-1926; (3) the study of the non-commutative algebras, their representations by linear transformations, and their application to the study of commutative number fields and their arithmetics.” As regards the first epoch, I have already written about the 1918 papers; the other dozen or so papers show her thinking developing from the old (19th century) ways of doing algebra and invariant theory to the new ideas of what Weyl calls the second epoch. She summarized her work in that first epoch in her Habilitation submission. I have included that here as Appendix B.

To summarize her work after 1919, let me begin by quoting the Russian topologist P. S. Alexandrov. “When we speak of Emmy Noether as a mathematician we mean not so much the early works but instead the period beginning about 1920 when she struck the way into a new kind of algebra.[She] herself is partly responsible for the fact that her work of the early period is rarely given the attention [among mathematicians] that it would naturally deserve: with the singlemindedness that was part of her nature, she herself was ready to forget what she had done in the early years of her scientific life, since she considered those results to have been a diversion from the main path of her research, which was the creation of a general, abstract algebra. It was she who taught us to think in terms of simple and general algebraic concepts - homomorphic mappings, groups and rings with operators, ideals ... theorems such as the ‘homomorphism and isomorphism theorems’, concepts such as the ascending and descending chain conditions for subgroups and ideals, or the notion of groups with operators were first introduced by Emmy Noether and have entered into the daily practice of a wide range of mathematical disciplines. ... We need only glance at Pontryagin’s work on the theory of continuous groups, the recent work of Kolmogorov on the combinatorial topology of locally compact spaces, the work of Hopf on the theory of continuous mappings, to say nothing of van der Waerden’s work on algebraic geometry, in order to sense the influence of Emmy Noether’s ideas. This influence is also keenly felt in

H. Weyl's book *Gruppentheorie und Quantenmechanik*." [29] All who have written about her recall that she always worked with a lively group of mathematicians around her. She gave lecture courses in Göttingen and elsewhere and loved to talk mathematics with groups of like-minded mathematicians. She had many very good students [4] and her influence extended well beyond her published papers. A notable example is given by Jacobson. "As is quite well known, it was Noether who persuaded P.S. Alexandrov and Heinz Hopf to introduce group theory into combinatorial topology and formulate the then existing simplicial homology theory in group-theoretic terms in place of the more concrete setting of incidence matrices." Alexandrov and Hopf say in the preface to their book *Topologie* (Berlin 1935) "Emmy Noether's general mathematical insights were not confined to her specialty - algebra - but affected anyone who came in touch with her work."

It was in the second epoch according to Weyl, 1920-26, that she founded the approach of modern abstract algebra. Jacobson describes how this came about; numbers refer to the list in Appendix A. "Abstract algebra can be dated from the publication of two papers by Noether, the first, a joint paper with Schmeidler, *Moduln in nichtkommutativen Bereichen* ... (no.17) and *Idealtheorie in Ringbereichen* (no.19). Of these papers, ..., the first is of somewhat specialized interest and its influence was negligible. Only in retrospect does one observe that it contained a number of important ideas whose rediscovery by others had a significant impact on the development of the subject. The truly monumental work *Idealtheorie in Ringbereichen* belongs to one of the mainstreams of abstract algebra, commutative ring theory, and may be regarded as the first paper in this vast subject..." Though the terminology - ideal theory, rings, Noetherian rings, the chain condition, etc. - is unfamiliar to most physicists, one can read Weyl's lucid account in his memorial address and gain some understanding of why Jacobson says "By now her contributions have become so thoroughly absorbed into our mathematical culture that only rarely are they specifically attributed to her."

In 1924 B. L. van der Waerden came to Göttingen having just finished his university course at Amsterdam. According to Kimberling, van der Waerden then mastered her the-

ories, enhanced them with findings of his own, and like no one else promulgated her ideas. [1] In her obituary, van der Waerden wrote that “her abstract, nonvisual conceptualizations met with little recognition at first. This changed as the productivity of her methods was gradually perceived even by those who did not agree with them. ... Prominent mathematicians from all over Germany and abroad came to consult with her and attend her lectures. ... And today, carried by the strength of her thought, modern algebra appears to be well on its way to victory in every part of the civilized world” [10] His book *Moderne Algebra*, as is credited on the title page, is based on the lectures of Emmy Noether and Emil Artin. According to Garrett Birkhoff, this book precipitated a revolution in the history of algebra. “Both the axiomatic approach and much of the content of ‘modern’ algebra dates back to before 1914. However, even in 1929 its concepts and methods were still considered to have marginal interest as compared with those of analysis... By exhibiting their mathematical and philosophical unity, and by showing their power as developed by Emmy Noether and her younger colleagues (most notably E. Artin, R. Brauer and H. Hasse), van der Waerden made ‘modern algebra’ suddenly seem central in mathematics. It is not too much to say that the freshness and enthusiasm of his exposition electrified the mathematical world.” [30] The first edition of *Moderne Algebra* was published in 1931. In the 1950’s when I was a graduate student in the University of Chicago, modern algebra certainly appeared central to us. Though we were graduate students studying physics, modern algebra was a subject we all aspired to learn. I believe it affected profoundly how modern physicists think and work.

The major papers in the third and final period, 1927-1935, are *Hyperkomplexe Grössen und Darstellungstheorie* (no.33), *Beweis eines Hauptsatzes in der Theorie der Algebren* (no.38), and *Nichtkommutative Algebren* (no.40). The reader is referred to Jacobson [3] for a detailed description of their content and significance from a contemporary point of view. Weyl says about the work of this period that “The theory of non-commutative algebras and their representations was built up by Emmy Noether in a new unified, purely conceptual manner by making use of all the results that had been accumulated by the ingenious labors of decades by Frobenius, Dickson, Wedderburn and others.” She found the idea

of automorphism useful, and made major contributions to cohomology theory. The work of this period is of great interest to present-day mathematicians, and theorists are finding it of value in their analyses of quantum field theories [27] and lattice gauge field theories [28]. It is also important, for example, in modern number theory. According to Jacobson, “of equal importance with [her] specific achievements were Noether’s contributions in unifying the field and providing the proper framework for future research.”

According to Weyl of her predecessors in algebra and number theory, Richard Dedekind was most closely related to her. She edited with Fricke and Ore the collected mathematical works of Dedekind, and the commentaries are mostly hers. She also edited the correspondence of Georg Cantor and Richard Dedekind. In addition to doing mathematics, giving lectures and lecture courses, supervising doctoral students and writing papers, Emmy Noether was a voluminous correspondent, especially with Ernst Fischer, a successor to Gordan in Erlangen; and H. Hasse, and was very active editing for *Mathematische Annalen*. [4]

The following tribute to Noether’s work was written by A. Einstein. “In the realm of algebra, in which the most gifted mathematicians have been busy for centuries, she discovered methods which have proved of enormous importance... Pure mathematics is, in its way, the poetry of logical ideas. One seeks the most general ideas of operation which will bring together in simple, logical and unified form the largest possible circle of formal relationships. In this effort toward logical beauty spiritual formulas are discovered necessary for the deeper penetration into the laws of nature.” [31]

V. BRIEF BIOGRAPHY

Emmy Noether was born Amalie Emmy Noether in Erlangen, Germany in 1882. Her father Max was a professor of mathematics in the university. She was born into a mathematical family. There were people of known mathematical ability on her grandmother’s side, and her younger brother Fritz became an applied mathematician. Because both father and daughter published papers frequently referred to in the mathematical literature, the work

done by Max is sometimes confused with that of his daughter. Max was a distinguished mathematician best known for the papers he published in 1869 and 1872. [32] This work was important for the development of algebraic geometry; it proved what the mathematicians call Noether's fundamental theorem, or the residue theorem. The theorem specifies conditions under which a given polynomial $F(x,y)$ can be written as a linear combination of two given polynomials f and g with polynomial coefficients. Hermann Weyl says about Max's work "...Clebsch had introduced Riemann's ideas into the geometric theory of algebraic curves and Noether became, after Clebsch had passed away young, his executor in this matter: he succeeded in erecting the whole structure of the algebraic geometry of curves on the basis of the so-called Noether residual theorem." [7] About the man he said "... such is the impression I gather from his papers and even more from the many obituary biographies [he wrote] a very intelligent, warm-hearted harmonious man of many-sided interests and sterling education." Max was successor to Felix Klein. Klein made Erlangen famous by announcing the Erlangen Program while he was professor there. The Erlangen Program was to classify and study geometries according to properties which remain invariant under appropriate transformation groups. With this program "various geometries previously studied separately were put under one unifying theory which today still serves as a guiding principle in geometry." [1] Klein left to join Hilbert in Göttingen, and Max Noether and Paul Gordan were the two Erlangen professors mainly responsible for the mathematical atmosphere in which Emmy grew up. Little has been written so far about Emmy's mother.

During most of the 19th century women were not allowed in European and North American universities and laboratories. The formal education of girls ended at age fourteen in Germany. However, as Emmy was growing up change was in the air. In 1898 the Academic Senate in the University of Erlangen declared that the admission of women students would "overthrow all academic order." [2] Nevertheless in 1900 Emmy got permission to attend lectures. The university registry shows then that two of 986 students attending lectures were female. However, women were not allowed to matriculate. Emmy attended lectures, and passed matura examinations at a nearby Gymnasium in 1903. In the winter she went

to Göttingen and attended lectures given by Schwarzschild, Minkowski, Klein, and Hilbert. Of course she was not allowed to enroll. In 1904 it became possible for females to enroll in the University of Erlangen and take examinations with the same rights as male students. She returned and did a doctoral thesis under the supervision of her father's friend and colleague Paul Gordan. The title of her thesis *On Complete Systems of Invariants for Ternary Biquadratic Forms*. It contains a tabulation of 331 ternary quartic covariant forms! It was officially registered in 1908. She was Gordan's only doctoral student. [4] She quickly moved on from this calculational phase to David Hilbert's more abstract approach to the theory of invariants. In a famous paper of 1888, Hilbert gave a proof by contradiction of the existence of a finite basis for certain invariants. It was the solution to a problem Gordan had worked on for many years and Gordan, after reading it, exclaimed, "Das ist nicht Mathematik; das ist Theologie." Gordan was an algebraist of the old school.

After obtaining her doctorate, Emmy Noether stayed in Erlangen in an unpaid capacity doing her own research, supervising doctoral students and occasionally substituting for her father at his lectures until 1915 when Hilbert invited her to join his team in Göttingen. This was the most active and distinguished center of mathematical research in Europe. However the mathematics faculty led by Hilbert and Klein found it impossible to obtain a university Habilitation for Emmy. Without that she could not teach or even give any University lectures. Her mathematical colleagues all supported her but at that time the Habilitation was awarded only to male candidates and Hilbert could not get around this. From 1916 to 1919, when finally she was given Habilitation, she often gave lectures which formally were Hilbert's; the lectures were advertised as *Mathematisch-Physikalisches Seminar*, [title], Professor Hilbert with the assistance of Fr. Dr. E. Noether. Finally awarded Habilitation, she could announce her own lectures. She remained, however, in an unpaid position, and it was not until 1923, when she was 41, that she was given a university position - but only that of *nicht-beamteter ausserordentlicher Professor*. The position carried with it no salary. However, Hilbert was able to arrange for her to have a *Lehrauftrag* for algebra which carried a small stipend.

In 1933 when the Nazi Party came to power, Jews were forced out of their academic positions by decree. The Nazis didn't want 'Jewish science' taught in the University. Emmy Noether was a Jewish woman and lost her position. At that time 3 of the 4 institutes of mathematics and physics were headed by Jews - Courant, Franck, and Born. They all had to leave their teaching positions. Hermann Weyl took over from Courant for a while thinking he could hold things together, that this was a transitory bad patch and that reason would prevail. Before a year was out he saw otherwise and also left Göttingen. He says of that period: "A stormy time of struggle like this one we spent in Göttingen in the summer of 1933 draws people closely together; thus I have a vivid recollection of these months. Emmy Noether - her courage, her frankness, her unconcern about her own fate, her conciliatory spirit - was in the midst of all the hatred and meanness, despair and sorrow surrounding us, a moral solace." Otto Neugebauer's photo of her at the railroad station leaving Göttingen in 1933 is shown here.

There were only two positions offered Noether in 1933 when she had to flee the Nazi's. One was a visiting professorship at Bryn Mawr supported, in part, by Rockefeller Foundation funds; and the other was in Somerville College, Oxford, where she was offered a stipend of fifty pounds aside from living accommodations. She went to Bryn Mawr. While there she was invited to give a weekly course of two hour lectures at the Institute for Advanced Study in Princeton. She traveled there by train each week to do so. Jacobson attended those lectures in 1935 and recalls that she announced a brief recess in her course because she had to undergo some surgery. Apparently the operation was followed by a virulent infection and she died quite unexpectedly. According to Weyl, "She was at the summit of her mathematical creative power" when she died.

Many people have written about how helpful and influential she was in the work of others. She not infrequently tended not to have her name included as author on papers to which she had contributed in order to promote the careers of younger people. She apparently was quite content with this and didn't feel a necessity to promote her own fame. She lived a very simple life and is reported to have been quite a happy person though she existed on

meager funds. Einstein wrote this tribute to her in his Letter to the Editor of the New York Times . [31] “ The efforts of most human beings are consumed in the struggle for their daily bread, but most of those who are, either through fortune or some special gift, relieved of this struggle are largely absorbed in further improving their worldly lot. Beneath the effort directed toward the accumulation of worldly goods lies all too frequently the illusion that this is the most substantial and desirable end to be achieved; but there is, fortunately, a minority composed of those who recognize early in their lives that the most beautiful and satisfying experiences open to humankind are not derived from the outside, but are bound up with the development of the individual’s own feeling, thinking and acting. The genuine artists, investigators and thinkers have always been persons of this kind. However inconspicuously the life of these individuals runs its course, none the less the fruits of their endeavors are the most valuable contributions which one generation can make to its successors.”

ACKNOWLEDGMENTS

It is a pleasure to express my gratitude to the organizers of the Conference for providing the occasion to write this paper, to Basil Gordon for illuminating mathematical discussions and close editing of the contents particularly as they deal with mathematics, and to Terry Tomboulis for helpful conversations.

APPENDIX A: PUBLICATION LIST

This list does not contain edited, and annotated, books and papers.

1. Über die Bildung des Formensystems der ternären bi-quadratischen Form. Sitz. Ber. d. Physikal.-mediz. Sozietät in Erlangen 39 (1907), pp. 176-179.
2. Über die Bildung des Formensystems der ternären bi-quadratischen Form. Journal f. d. reine u. angew. Math. 134 (1908), pp. 23-90.
3. Zur Invariantentheorie der Formen von n Variablen. J. Ber. d. DMV 19 (1910), pp. 101-104.
4. Zur Invariantentheorie der Formen von n Variablen. Journal f. d. reine u. Angew. Math. 139 (1911), pp. 118-154.
5. Rationale Funktionenkörper. J. Ber. d. DMV 22 (1913), pp. 316-319.
6. Körper und Systeme rationaler Funktionen. Math. Ann. 76 (1915), pp. 161-196.
7. Der Endlichkeitssatz der Invarianten endlicher Gruppen. Math. Ann. 77 (1916), pp. 89-92.
8. Über ganze rationale Darstellung der Invarianten eines Systems von beliebig vielen Grundformen. Math. Ann. 77 (1916), pp. 93-102. (cf. No. 16).
9. Die allgemeinsten Bereiche aus ganzen transzendenten Zahlen. Math. Ann. 77 (1916), pp. 103-128. (cf. No. 16)
10. Die Funktionalgleichungen der isomorphen Abbildung. Math. Ann. 77 (1916), pp. 536-545.
11. Gleichungen mit vorgeschriebener Gruppe. Math. Ann 78 (1918), pp. 221-229. (cf. No. 16).
12. Invarianten beliebiger Differentialausdrücke. Nachr. v. d. ges. d. Wiss. zu Göttingen 1918. pp. 37-44.
13. Invariante Variationsprobleme. Nachr. v. d. Ges. d. Wiss. zu Göttingen 1918, pp. 235-257.
14. Die arithmetische Theorie der algebraischen Funktionen einer Veränderlichen in ihrer Beziehung zu den übrigen Theorien und zu der Zahlkörpertheorie. J. Ber. d. DMV 28 (1919), pp. 182-203.
15. Die Endlichkeit des Systems der ganzzahligen Invarianten binärer Formen. Nachr. v. d. Ges. d. Wiss. zu Göttingen 1919, pp. 138-156.
16. Zur Reihenentwicklung in der Formentheorie. Math. Ann. 81 (1920), pp. 25-30.
17. Moduln in nichtkommutativen Bereichen, insbesondere aus Differential- und Differenzenausdrücken. Co-authored by W. Schmeidler. Math. Zs. 8 (1920), pp. 1-35.
18. Über eine Arbeit des im Kriege gefallenen K. Hentzelt zur Eliminationstheorie. J. Ber. d. DMV 30 (1921), p. 101
19. Idealtheorie in Ringbereichen Math. Ann. 83 (1921), pp. 24-66.
20. Ein algebraisches Kriterium für absolute Irreduzibilität. Math Ann. 85 (1922), pp. 26-33.
21. Formale Variationsrechnung und Differentialinvarianten. Encyklopädie d. math. Wiss. III, 3 (1922), pp. 68-71 (in: R. Weitzenböck, Differentialinvarianten).
22. Bearbeitung von K. Hentzelt: Zur Theorie der Polynomideale und Resultanten. Math. Ann. 88 (1923), pp. 53-79.
23. Algebraische und Differentialvarianten. J. Ber. d. DMV 32 (1923), pp. 177-184.
24. Eliminationstheorie und allgemeine Idealtheorie. Math. Ann. 90 (1923), pp. 229-261.
25. Eliminationstheorie und Idealtheorie. J. Ber. d. DMV 33 (1924), pp. 116-120.
26. Abstrakter Aufbau der Idealtheorie im algebraischen Zahlkörper. J. Ber. d. DMV 33 (1924), p. 102.
27. Hilbertsche Anzahlen in der Idealtheorie. J. Ber. d. DMV 34 (1925), p. 101
28. Gruppencharaktere und Idealtheorie. J. Ber. d. DMV 34 (1925), P. 144.
29. Der Endlichkeitssatz der Invarianten endlicher linearer Gruppen der Charakteristik p . Nachr. v. d. Ges. d. Wiss. zu Göttingen 1926, pp. 28-35.
30. Abstrakter Aufbau der Idealtheorie in algebraischen Zahl- und Funktionenkörpern. Math. Ann. 96 (1927), pp. 26-61.
31. Der Diskriminantensatz für die Ordnungen eines algebraischen Zahl- oder Funktionenkörpers. Journal f. d. reine u. angew. Math. 157 (1927), pp. 82-104.
32. Über minimale Zerfällungskörper irreduzibler Darstellungen. Co-authored by R. Brauer. Sitz. Ber. d. Preuss. Akad. d. Wiss. 1927, pp. 221-228
33. Hyperkomplexe Größen und Darstellungstheorie in arithmetischer Auffassung. Atti Congresso Bologna 2 (1928), pp. 71-73.
34. Hyperkomplexe Größen und Darstellungstheorie. Math. Zs. 30 (1929), pp. 641-692.
35. Über Maximalbereiche aus ganzzahligen Funktionen. Rec. Soc. Math. Moscou 36 (1929), pp. 65-72.
36. Idealdifferentiation und Differenten. J. Ber. d. DMV 39 (1930), p. 17.
37. Normalbasis bei Körpern ohne höhere Verzweigung. Journal f. d. reine u. angew. Math. 167 (1932), pp. 147-152.

38. Beweis eines Hauptsatzes in der Theorie der Algebren. Co-authored by R. Brauer and H. Hasse. *Journal f. d. reine u. angew. Math.* 167 (1932), pp. 399-404.
39. Hyperkomplexe Systeme in ihren Beziehungen zur kommutativen Algebra und zur Zahlentheorie. *Verhandl. Intern. Math.-KongreßZürich 1* (1932), pp. 189-194.
40. Nichtkommutative Algebren. *Math. Zs.* 37 (1933), pp. 514-541.
41. Der Hauptgeschlechtssatz für relativ-galoissche Zahlkörper. *Math. Ann.* 108 (1933), pp. 411-419.
42. Zerfallende verschränkte Produkte und ihre Maximalordnungen. *Actualités scientifiques et industrielles* 148 (1934) (15 pages).
43. Idealdifferentiation und Differenten. *Journal f. d. reine u. angew. Math.* 188 (1950), pp. 1-21.

APPENDIX B: EXCERPT FROM HABILITATION SUBMISSION.

Noether characterized her published papers from the period 1907 to 1918 in her submission for Habilitation. The submission reads in part (number insertions refer to the list of publications in Appendix A):

“My dissertation and a later paper ... belong to the theory of formal invariants, as was natural for me as a student of Gordan. The lengthiest paper, ‘Fields and Systems of Rational Functions’ (6) concerns questions about general bases; it completely solves the problem of rational representation and contributes to the solution of other finiteness problems. An application of these results is contained in ‘The Finiteness Theorem for Invariants of Finite Groups’ (7) which offers an absolutely elementary proof by actually finding a basis. To this line of investigation also belongs the paper ‘Algebraic Equations with Prescribed Group’ (11) which is a contribution to the construction of such equations for any field range.... The paper ‘Integral Rational Representation of Invariants’ (8) proves valid a conjecture of D. Hilbert ... With these wholly algebraic works belong two additional works ‘A Proof of finiteness for Integral Binary Invariants’ (15) ... and an investigation with W. Schmeidler of noncommutative one-sided modules... ‘Alternatives with Nonlinear Systems of Equations’... The longer work ‘The Most General Ranges of Completely Transcendental Numbers’ (9) uses along with algebraic and number-theoretic techniques some abstract set theory ...In this same direction is the paper ‘Functional Equations and Isomorphic Mapping’ (10) which yields the most general isomorphic mapping of an arbitrarily abstractly defined field. Finally, there are two works on differential invariants and variation problems (12,13)...” [4]

APPENDIX C: TITLES FROM A RECENT ISSUE OF CURRENT CONTENTS

1. V. Iyer and R.M. Wald, *Some Properties of the Noether Charge and a Proposal for Dynamical Black Hole Entropy*, Phys. Rev. D **50**, 846, 1994.
2. O. Castanos, R. Lopezpena and V. I. Manko, *Noether Theorem and Time-dependent Quantum Invariants*, J. Phys. A: Mathematical and General, **27**, 1751 (1994).
3. M. Forger, J. Laartz and U. Schaper, *The Algebra of the Energy-momentum Tensor and the Noether Currents in Classical Non-linear Sigma Models*, Commun. Math. Phys. **159**, 319 (1994).
4. P. G. Henriques, *The Noether Theorem and the Reduction Procedure for the Variational Calculus in the Context of Differential Systems*, Comptes Rendus De L Academie Des Sciences Serie I-Mathematique **317**, 987 (1993).
5. R. M. Wald, *Black Hole Entropy is the Noether Charge*, Phys. Rev. D **48** N8:R3427 (1993).
6. Z. P. Li, *Generalized Noether Theorem and Poincare-Cartan Integral Invariant for Singular High-order Lagrangian in Fields Theories*, Science in China Series-A-Mathematics Physics Astronomy Technological Sciences **36**, 1212 (1993).
7. D. E. Neuenschwander and S. R. Starkey, *Adiabatic Invariance Derived from the Rund-Trautman Identity and Noether Theorem*, Am. J. Phys. **61**, 1008 (1993).
8. L. Lusanna, *The Shanmugadhasan Canonical Transformation, Function Groups and the Extended 2nd Noether Theorem*, International Journal of Modern Physics A, **8**, 4193 (1993).
9. A. Weldo, *Hard Thermal Loops and their Noether Currents*, Can. J. Phys. **71**, N5-6:300 (1993).
10. A. Milinski, *Skolem-Noether Theorems and Coalgebra Actions*, Communications in Algebra **21**, 3719 (1993).
11. Z. P. Li, *Noether Theorem and its Inverse Theorem in Canonical Formalism for Nonholonomic Nonconservative Singular System*, Chinese Science Bulletin **38**, 1143 (1993).
12. F. M. Mahomed, A. H. Kara and P. G. L. Leach, *Lie and Noether Counting Theorems for One-Dimensional Systems*, Journal of Mathematical Analysis and Applications **178** 116 (1993). N1:116-129.
13. G. Reinish and J. C. Fernandez, *Noether Theorem and the Mechanics of Nonlinear Solitary Waves*, Phys. Rev. B: Condensed Matter **48** 853 (1993).
14. A. I. Tuzik, *On the Noether Conditions of One Dual Discreet Equation of Convolution Type with almost Stabilizing Multipliers*, Dokl. Akad. Nauk Belarusi, **37** 118 (1993).
15. V. A. Dorodnitsyn *The Finite-Difference Analogy of Noether Theorem*, Dokl. Akad. Nauk, **328** 678 (1993).
16. M. Forger and J. Laartz *The Algebra of the Energy-Momentum Tensor and the Noether Currents in Off-critical WZNW Models*, Modern Physics Letters A., **8** 803 (1993).
17. X. C. GAO, J. B. Xu, T. Z. Qian and J. Gao *Quantum Basic Invariants and Classical Noetherian Theorem*, Physica Scripta, **47** 488 (1993).
18. N. A. Lemos *Symmetries, Noether Theorem and Inequivalent Lagrangians Applied to Nonconservative Systems*, Revista Mexicana De Fisica, **39** 304 (1993).
19. J. L. Colliotthelene *The Noether-Lefschetz Theorem and Sums of 4 Squares in the Rational Function Field $R(X, Y)$* , Compositio Mathematica, **86** 235 (1993).
20. S. Caenepeel *Computing The Brauer-Long Group of a Hopf Algebra. 2. The Skolem-Noether Theory*, Journal of Pure and Applied Algebra, **84** 107 (1993).
21. Z. P. Li *Generalized Noether Theorems in Canonical Formalism for Field Theories and their applications*, Int. J. Theor. Phys. **32** 201 (1993).
22. O. Castanos and R. Lopezpena *Noether Theorem and Accidental Degeneracy*, J. Phys. A: Mathematical and General **25** 6685 (1992).
23. X. Gracia and J. M. Pons *A Hamiltonian Approach to Lagrangian Noether Transformations*, J. Phys. A: Mathematical and General **25** 6357 (1992).
24. K. Kurano *Noether Normalizations for Local Rings of Algebraic Varieties*, Proceedings of American Mathematical Society **116** 905 (1992).
25. L. B. Szabados *On Canonical Pseudotensors, Sparling form and Noether Currents*, Classical and Quantum Gravity **9** 2521 (1992).
26. E. Gonzalezacosta and M. G. Coronagalindo *Noether Theorem and the Invariants for Dissipative and Driven Dissipative Like Systems*, Revista Mexicana De Fisica **38** 511 (1992).
27. S. L. Guo *Noether (Artin) Classical Quotient Rings of Duo Rings and Projectivity and Injectivity of Simple Modules over Duo Rings*, Chinese Science Bulletin **37** 877 (1992).
28. X. C. Gao , J. B. Xu and J. Gao *A New Approach to the Study of Noether Invariants and Symmetry Algebras*, Prog. Theor. Phys. **87** 861 (1992).
29. Z. P. Li *Noether Theorems in Canonical Formalism and their Applications*, Chinese Science Bulletin **37** 258 (1992).
30. J. Krause *Some Remarks on the Generalized Noether Theory of Point Summetry Transformations of the Lagrangian*, J. Phys. A: Mathematical and General **25** 991 (1992).

31. M. Kobayashi *On Noether Inequality for Threefolds*, Journal of the Mathematical Society of Japan **44** 145 (1992).
32. S. K. Luo *Generalized Noether Theorem for Variable Mass Higher-order Nonholonomic Mechanical Systems in Noninertial Reference Frames*, Chinese Science Bulletin **36** 1930 (1991).
33. G. Dzhangibekov *On the Conditions of Noether Property and Index of some 2-Dimensional Singular Integral Operators*, Dokl. Akad. Nauk SSSR **319** 811 (1991).
34. R. D. Sorkin *The Gravitational Electromagnetic Noether Operator and the 2nd-Order Energy Flux*, Proc. R. Soc. London Ser. A. **435** 635 (1991).
35. C. Cilibeto and A. F. Lopez *On the Existence of Components of the Noether-Lefschetz Locus with Given Codimension*, Manuscripta Mathematica **73** 341 (1991).
36. J. F. Carinena, Fernandeznunez and E. Martinez *A Geometric Approach to Noether 2nd Theorem in Time-Dependent Lagrangian Mechanics*, Letters in Mathematical Physics **23** 51 (1991).
37. V. G. Kravchenko and G. S. Litvinchuk *Noether Theory of Unbounded Singular Integral Operators with a Carleman Shift*, Mathematical Notes **49** 45 (1991).
38. M. Koppinen *A Skolem-Noether Theorem for Coalgebra Measurements*, Archiv Der Mathematik **57** 34 (1991).
39. I. C. Moreira and M. A. Almeida *Noether Symmetries and the Swinging Atwood Machine*, Journal De Physique II. **1** 711 (1991)
40. B. Y. Shteinberg, *The Noether Property and the Index of Multidimensional Convolutions with Coefficients of Fast Oscillating Type*, Siberian Mathematical Journal **31** 682 (1990).
41. L. Lusanna *The 2nd Noether Theorem as the Basis of the Theory of Singular Lagrangians and Hamiltonians Constraints*, Rivista Del Nuovo Cimento **14** 1 (1990).
42. S. O. Kim *Noether-Lefschetz Locus for Surfaces*, Transactions of the American Mathematical Society **324** 369 (1991).
43. H. A. Alkuwari and M. O. Taha *Noether Theorem and Local Gauge Invariance*, Am. J. Phys. **59** 363 (1991).
44. F. A. Lunev *Analogue of Noether Theorem for Non-Noether and Nonlocal Symmetries*, Theoretical and Mathematical Physics **84** 816 (1990).
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