Holographic description of boundary and interface CFTs

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Outline

- Introduction
- Six dimensional supergravity
- Half BPS solutions
- Holographic Interface CFTs
- Generalized solutions: Holographic Boundary CFTs
- Discussion
based on the following two papers:

1107.1722  M. Chiodaroli, E. D’Hoker, Y. Guo and M.G.
1111.6912  M. Chiodaroli, E. D’Hoker and M.G.

and work in progress together with M. Chiodaroli and E. D’Hoker.
A boundary CFT is a CFT living in a space with boundary (i.e. the half plane) boundary conditions preserve a single copy of the \( \text{Virasoro} \otimes \text{Virasoro} \) conformal symmetry of the CFT in the bulk.

Modular transformation maps half plane to semi-infinite cylinder. Boundary conditions are imposed by boundary state.

Applications: Open string vacua, D-branes, condensed matter and stat-mech etc.

\[
T_{xy} = 0 \leftrightarrow T(z) = \bar{T}(\bar{z}) \\
(L_n - \bar{L}_{-n}) \mid B \rangle = 0
\]
A conformal interface is a junction between two CFTs preserving part of the conformal symmetry.

Gluing conditions need to be scale invariant. In 2d continuity of $T_{xy}$ across the interface is sufficient.

The folding trick relates interface CFT to boundary CFT: conformal boundary conditions in tensor product $CFT_1 \otimes CFT_2$. Affleck and Ludwig; Bachas, de Boer, Dijkgraaf and Ooguri
A simple toy model is given by a compact boson $\phi \sim \phi + 2\pi$ with a jump in the coupling/tension

$$S = 2r_+^2 \int_{x<0} dx \, dy \, \partial_a \phi \partial^a \phi + 2r_-^2 \int_{x>0} dx \, dy \, \partial_a \phi \partial^a \phi$$

After rescaling the boson has a jump in the radius $\phi \sim \phi + 2\pi r_\pm$

After folding two bosons $\phi_1, \phi_2$ different radii conformal boundary conditions correspond to a diagonal D1 brane in a 2 torus spanned by $\phi_1, \phi_2$ with radius $r_\pm$.

Calculate g-factor $g = \langle 0 \mid B \rangle$ or boundary entropy $S_{bd} = \ln g$ of the interface
Interfaces between 2 CFTs can be generalized to junctions of 3 or more CFTs, for example a star graph.

- Conformal boundary conditions: $\sum_i T^{(i)}_{xy} = 0$.
- Example: $n$ bosons $\phi_i$ with radius $r_i$ on half spaces, joined at interface.
- Conformal boundary conditions: $p$ dim D brane in $n$-torus g-factors calculated in BCFT.
- Reminiscent of $(p, q)$-string junctions.
- Boundary entropy can be calculated as before.
The Janus solution is a simple example of a holographic realization of an interface CFT. Consider $d+1$ dim AdS gravity and massless scalar (dilaton).

AdS$_d$ slicing of AdS$_{d+1}$ preserves SO$(2,d-1)$ of the SO$(2,d)$ conformal symmetry.

$$ds^2 = dx^2 + f(x)ds^2_{AdS_d}, \quad \phi = \phi(x)$$

AdS vacuum: $f(x) = \cosh^2 x$, $\phi(x) = \phi_0$. Janus solution: As $x \to \pm \infty \phi \to \phi_{\pm}$

asymptotic form of metric as $x \to \pm \infty$ (AdS$_d$ in Poincare coord, $\xi \in [0,\infty]$).

$$ds^2 = dx^2 + \frac{e^{2|x|}}{\xi^2} \left( d\xi^2 - dt^2 - d\vec{y}_{d-2}^2 \right) + o(1)$$

Three boundary components: $x \to \pm \infty$ (two d-dim half spaces), $\xi \to 0$ (a d-1 dimensional interface).

Dilaton is dual to coupling constant of CFT. Interface CFT with jumping coupling.
For $AdS_3$ it is possible to relate the boundary entropy to entanglement entropy by Cardy:

$$S_A = \frac{c}{6} \log \frac{L}{\epsilon} + \log g_B$$

Holographic calculation of entanglement entropy by Ruy and Takayanagi.

When embedded into supergravity, the Janus solution breaks all super symmetries (since $\delta \lambda \neq 0$ for all $\epsilon$).

**Goals**

- Can the Janus solution be generalized to preserve supersymmetries?
- Can one find holographic duals of junctions of CFTs?
- Can one find holographic duals of boundary CFTs?
We consider the so called \((0,4)\) or type 4B supergravity in 6 dimensions. \((\text{Townsend; Romans})\). It has the following properties:

- It has 16 chiral supersymmetries
- It can be obtained by compactifying type IIB on \(K3\).
- The scalar fields parameterize the coset \(SO(5,21)/SO(5) \times SO(21)\).
- The field content is given by

<table>
<thead>
<tr>
<th>Field</th>
<th>(g_{\mu \nu})</th>
<th>(B^I_{\mu \nu})</th>
<th>(B^{R}_{\mu \nu})</th>
<th>(V^i_R)</th>
<th>(\psi^\alpha_{\mu})</th>
<th>(\chi^{r\alpha})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(SO(5))</td>
<td>1</td>
<td>5</td>
<td>1</td>
<td>5</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>(SO(21))</td>
<td>1</td>
<td>1</td>
<td>21</td>
<td>21</td>
<td>1</td>
<td>21</td>
</tr>
</tbody>
</table>

- The supersymmetry transformations are given by

\[
\delta \psi^\alpha_{\mu} = D_\mu \epsilon^\alpha - \frac{1}{4} H^I_{\mu \nu \rho} \gamma^{\nu \rho} (\Gamma^I)^{\alpha}_{\beta} \epsilon_{\beta}
\]

\[
\delta \chi^{r\alpha} = \frac{1}{\sqrt{2}} \gamma^\mu p^{IR}_\mu (\Gamma^I)^{\alpha}_{\beta} \epsilon_{\beta} + \frac{1}{12} \gamma^{\mu \nu \rho} H^{R}_{\mu \nu \rho} \epsilon^\alpha
\]
Probe branes in $AdS_3 \times S^3$

- Vacua: 6 dim Minkowski and $AdS_3 \times S_3$ supported by self-dual 3 form $H^i = \ast H^i$.
- $AdS_3 \times S_3$ is the near horizon limit of self-dual string solution in 6 dimensions.
- From 10 dim point of view: D5/D1 bound state wrapped on K3.
- AdS vacuum preserves 16 super symmetries. Half BPS probe branes preserve 8. $\kappa$-symmetry can be used to find all of these:

<table>
<thead>
<tr>
<th>probe brane</th>
<th>$AdS_3$</th>
<th>$S_3$</th>
<th>K3</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>$AdS_2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D3</td>
<td>$AdS_2$</td>
<td>$S_2$</td>
<td></td>
</tr>
<tr>
<td>D5</td>
<td>$AdS_2$</td>
<td></td>
<td>K3</td>
</tr>
<tr>
<td>D7</td>
<td>$AdS_2$</td>
<td>$S_2$</td>
<td>K3</td>
</tr>
<tr>
<td>D3</td>
<td>$AdS_2$</td>
<td></td>
<td>$C_2$</td>
</tr>
</tbody>
</table>
Half BPS solutions dual to interface or junction CFTs

- **Janus ansatz**: $AdS_2$ slicing of $AdS_3$.
- Global super algebra of vacuum $PSU(1,1|2) \times PSU(1,1|2)$ reduced to $PSU(1,1|2)$. Isometries reduced from $SO(2,2) \times SO(4)$ to $SO(2,1) \times SO(3)$.
- **Ansatz**: $AdS_2 \times S_2$ fibration over 2dim Riemann surface $\Sigma$
  \[
ds^2 = f_1^2 ds^2_{AdS_2} + f_2^2 ds^2_{S^2} + \rho^2 (dx^2 + dy^2)\]
- $\Sigma$ has boundary where $\text{vol}(S_2) \to 0$. Scale factor of $AdS_2$ diverges at isolated points at boundary $\partial \Sigma$. 
Ansatz

- For $AdS_3 \times S^3$ vacuum $\Sigma$ is the strip, $\Sigma = \{x + iy, x \in \mathbb{R}, y \in [0, \pi]\}$
- metric is given by

$$ds^2 = f_1^2 ds_{AdS_2}^2 + f_2^2 ds_{S^2}^2 + \rho^2 (dx^2 + dy^2)$$

with

$$f_1^2 = \cosh^2 x, \quad f_2^2 = \sin^2 (y), \quad \rho^2 = 1$$
Ansatz

Ansatz for other bosonic fields which depend on coordinates of $\Sigma$ and respect the $AdS_2 \times S_2 \times \Sigma$ fibration.

- Anti-symmetric tensor 2 form potentials

$$B^A = \Psi^A \omega_{AdS_2} + \Phi^A \omega_{S_2}, \quad A = 1, 2, \cdots, 26$$

- The scalar fields take values in the coset space $SO(5,m)/(SO(5) \times SO(m))$ via a frame field $V$

$$\eta = \begin{pmatrix} I_5 \\ -I_{21} \end{pmatrix}, \quad V = V^{(i,r)}_A = \begin{pmatrix} V^i_I & V^i_R \\ V^r_I & V^r_R \end{pmatrix}$$

- $A = (I,R)$ labels the fundamental representation of $SO(5,m)$. $(i,r)$ label the fundamental representation of $SO(5) \times SO(m)$, with $I,i = 1, \cdots, 5$ and $R,r = 6, \cdots, m+5$.

- The coset representative satisfy

$$\eta = V^t \eta V$$
Solving the BPS equations

General strategy for solving the BPS equation (Details can be found in arXiv:1107.1722).

Find nontrivial solution of

\[ \delta \chi^{R\alpha} = 0, \quad \delta \psi^{\alpha}_{\mu} = 0 \]

For some infinitesimal susy transformation parameter \( \epsilon^{\alpha} \).

- Bosonic fields which respect \( AdS_2 \times S_2 \times \Sigma \) fibration
- Expand susy parameter \( \epsilon \) in terms of a basis of Killing spinors on \( AdS_2 \) and \( S_2 \).
- Reduced BPS equations: Algebraic projector equations and diff. equations on \( \Sigma \).
- Projection conditions can be solved by choosing a \( SO(5) \times SO(21) \) gauge. Sets some scalars and AST to zero.
- Differential equation equation can be solved in terms of harmonic function \( H \).
- Solve Bianchi identities in terms of meromorphic function \( \lambda^A \).
- Check that equations of motion are satisfied.
General local solution

The general local solution of the BPS equations is given by harmonic and meromorphic functions

- A real positive harmonic function $H$ on $\Sigma$
- meromorphic functions $\lambda^A$, $A = 1, 2, \ldots, 26$
- Projection conditions set $\lambda^3 = \lambda^4 = \lambda^5 = 0$
- The meromorphic functions satisfy constraints on $\Sigma$

$$\lambda \cdot \bar{\lambda} = (\lambda^1)^2 + \cdots (\lambda^5)^2 - (\lambda^6)^2 - \cdots (\lambda^{26})^2 = 2, \quad \bar{\lambda} \cdot \lambda \geq 2$$

- All bosonic fields can be expressed in terms of $H, \lambda^A$. Metric factors:

$$f_1^4 = H^2 \frac{\bar{\lambda} \cdot \lambda + 2}{\bar{\lambda} \cdot \lambda - 2}$$
$$f_2^4 = H^2 \frac{\bar{\lambda} \cdot \lambda - 2}{\bar{\lambda} \cdot \lambda + 2}$$
$$\rho^4 = \frac{|\partial_w H|^4}{16H^2} (\bar{\lambda} \cdot \lambda + 2)(\bar{\lambda} \cdot \lambda - 2)$$
Scalar coset in gauge fixed form: \( i, I = 1, 2, r, R = 6, 7, \cdots 26 \).

\[
V = \begin{pmatrix}
V_i^I & 0 & V_i^R \\
0 & I_3 & 0 \\
V_r^I & 0 & V_r^R
\end{pmatrix}
\]

The combinations \( V^\pm_{l,R} = V_{l,R}^1 \pm iV_{l,R}^2 \) can be expressed as

\[
V^+_A = X(\bar{\lambda}_A - |X|^2\lambda_A)/(1 - |X|^4)
\]
\[
V^-_A = \bar{X}(\lambda_A - |X|^2\bar{\lambda}_A)/(1 - |X|^4)
\]

where \( |X|^2 + 1/|X|^2 = \bar{\lambda} \cdot \lambda \).

AST potential \( B^3 = B^4 = B^5 = 0 \) and

\[
\Phi^A = -\sqrt{2} \frac{H \text{Re}(\lambda^A)}{\bar{\lambda} \cdot \lambda + 2} + \Phi^A
\]
\[
\Psi^A = -\sqrt{2} \frac{H \text{Im}(\lambda^A)}{\bar{\lambda} \cdot \lambda - 2} + \Psi^A
\]

\[
\Phi^A = \frac{1}{2\sqrt{2}} \int^w \partial_w H \lambda^A + \text{c.c.}
\]
\[
\Psi^A = \frac{i}{2\sqrt{2}} \int^w \partial_w H \lambda^A + \text{c.c.}
\]
The local solution solves BPS equations, Bianchi identities and equations of motion. However the metric and other fields are regular only if the following **regularity conditions** are satisfied:

- In the interior of $\Sigma$ we have $H > 0$ and $\bar{\lambda} \cdot \lambda > 2$;
- On the boundary $\partial \Sigma$ of $\Sigma$ we have $H = 0$ and $\text{Im}(\lambda^A) = 0$, except at isolated points;
- The one-forms $\lambda^A \partial_w H$ are holomorphic and nowhere vanishing in the interior of $\Sigma$, forcing the poles of $\lambda^A$ to coincide with the zeros of $\partial_w H$;
- The functions $\lambda^A$ are holomorphic near $\partial \Sigma$, thereby allowing for poles in $\lambda^A \partial_w H$ on $\partial \Sigma$ only at those points where $\partial_w H$ has a pole.
The constraint and regularity conditions involving $\lambda \cdot \lambda$ are conveniently solved by a light cone parameterization singling out the $A = 2$ and $A = 6$ direction. $L^A$ are a new set of meromorphic functions

$$\lambda^A = \frac{\sqrt{2} L^A}{L^6}$$

$$\lambda^2 = \frac{1}{L^6} \left( \frac{1}{2} - L^1 L^1 + L^R L^S \delta_{RS} \right)$$

$$\lambda^6 = \frac{1}{L^6} \left( -\frac{1}{2} - L^1 L^1 + L^R L^S \delta_{RS} \right)$$

where $A = 1, 7, 8, \cdots, 26$, and $R, S = 6, 7, 8, \cdots, 26$. The regularity condition $\text{Im}(\lambda^A) = 0$ on $\partial \Sigma$ implies

$$\text{Im}(L^A) = 0, \quad \text{for } w \in \partial \Sigma$$
Regular Solution on the half plane

The simplest choice of $\Sigma$ is the upper half plane where $\partial \Sigma$ is the real line.

- Conditions on harmonic function $H$: $N$ simple poles on the real line

$$H(w, \bar{w}) = \sum_{n=1}^{N} \left( \frac{i c_n}{w - x_n} - \frac{i c_n}{\bar{w} - x_n} \right)$$

- $L^A$ satisfying boundary conditions can be given by a sum of $P$ auxiliary poles on the real line

$$L^A(w) = l^A_\infty + \sum_{p=1}^{P} \frac{l^A_p}{w - y_p}$$
For regular solutions, avoiding curvature singularities requires $L^6$ and $\partial_w H$ to have common zeros. Therefore, we take $L^6$ to have the form,

$$L^6(w) = i l_\infty \prod_{n}^{N} \frac{(w - x_n)^2}{\prod_{p}^{P} (w - y_p)} \partial_w H(w)$$

To avoid an auxiliary pole or an extra zero at infinity, we need the number of auxiliary poles to be related to the number of physical poles

$$P_{disk} = 2N - 2$$

Regularity of the $\lambda^A$ at the auxiliary poles implies

$$l^1_p = \sqrt{\sum_{R} l^R_p l^R_p}$$
Near a pole of $H$, the metric takes an asymptotic $AdS_3 \times S^3$ form: $w = x_n + re^{i\theta}$ with $r \to 0$ asymptotics.

$$ds^2 \sim \sqrt{2\mu_n \cdot \mu_n} \left( \frac{dr^2}{r^2} + \frac{2c_n^2}{\mu_n \cdot \mu_n} \frac{1}{r^2} ds_{AdS_2}^2 + d\theta^2 + \sin^2 \theta \, ds_{S^2}^2 \right) + o(r^2)$$

The $\mu^A$ are directly related to the conserved charges on the asymptotic $S^3$

$$Q_n^A \equiv \int_{S^3} dB^A = i \sqrt{2\pi} \int_{x_n} \lambda^A \partial_w H + c.c. = 2 \sqrt{2\pi} \mu_n^A$$
Holographic interface solutions

- N=2 is minimal because of charge conservation, also P=2

\[
H(w, \bar{w}) = \left( \frac{ic_1}{w-x_1} + \frac{ic_2}{w-x_2} \right) + c.c, \quad L_A^A(w) = l_\infty^A + \frac{l_1^A}{w-y_1} + \frac{l_2^A}{w-y_2}
\]

- Half plane can be mapped to the strip via \( z = e^{iw} \).

- Conserved charge \( Q^A \) is charge carried by self dual string in 6 dim. Central charge of dual CFT \( c \sim Q_n \cdot Q_n \)

- as \( \text{Re}(w) \to \pm \infty \) scalars approach \( V^I_A \to (V^I_A)_\pm \)

  (a) Subset of scalars are "fixed" whose limit \( \text{Re}(w) \to \pm \infty \) is determined by the charge vector \( Q^A \) (attractor mechanism), i.e \( (V^I_A)_+ = (V^I_A)_- \)

  (b) The other scalars are "free" and can jump as \( \text{Re}(w) \to \pm \infty \), i.e \( (V^I_A)_+ = (V^I_A)_- \)
Holographic interface solutions

- Consider the truncation to only two scalars ("dilaton" and "axion")

"dilaton" $\phi$ is dual to dimension $(h, \bar{h}) = (1, 1)$ operator $O_\phi$ deformation of the dual CFT by source which jump across the interface

$$S_{\text{CFT}} \rightarrow S_{\text{CFT}} + c_+ \int_{x<0} dx \, dy \, O_\phi + c_- \int_{x>0} dx \, dy \, O_\phi$$

Holographic junction solutions

- A solution where $H$ has $N$ poles has $N$ asymptotic AdS regions, corresponding to $N$ half spaces in the dual CFT.
- Each asymptotic AdS space is characterized by the charge $Q^A$ and the value of the free (i.e. non attracted) scalars.

![Diagram showing holographic junction solutions](image)

- Exact match between parameters of solution ($x_i, y_j, c_i, I_k^A$ etc.) and charges $Q_k^A$ and values of free scalars $V_{iR}^k$ for each asymptotic AdS region.
- Each asymptotic AdS region dual to a CFT defined by the near horizon limit of a self-dual string with charge $Q^A$, deformed by operator source dual to unattracted scalars.
Holographic junction solutions

- Holographic dual: Junction of N CFTs (defined by charge $Q$) living on half-spaces glued together at a one dimensional interface. (See figure for the case $N=3$).
- Near Horizon limit of a half-BPS junction of N self dual strings in six dimensions.

Figure 2: (a) Three string junction (b) multi-string junction
Since the charge $Q^A$ is conserved, it seems impossible to have a solution with a single pole in $H$.

Such a solution would only have one asymptotic AdS region, i.e. would be dual to a CFT living on a single half space: Boundary CFT.

**Question**

Can one find holographic duals of boundary CFTs in our class of solutions?
AdS-cap

- The charge vector $Q^A_i$ determines the central charge of the dual CFT

$$c_i = \frac{3 Q_i \cdot Q_i}{16\pi^2 G_N}$$

- Charge conservation implies

$$\sum_i Q^A_i = 0$$ (1)

- We can consider charges for $Q \cdot Q = 0$. These leads to a singular solution (since $c = 0$ this implies $R_{AdS} = 0$) and call it ”AdS-cap”.

- Albeit singular some calculations like the holographic boundary entropy still make sense, giving a realization of a BCFT.

- $Q \cdot Q = 0$ means that we do not have a self dual string but a pure D1 or D5 brane.
Higher genus solutions

- It’s natural to consider a Riemann surface with more than one boundary. The simplest case is the annulus or cylinder.

\[ \Sigma = \left\{ w \in \mathbb{C}, \ 0 \leq Re(w) < 1, \ 0 \leq Im(w) \leq \frac{t}{2} \right\}, \quad w \sim w + 1 \]  

- The harmonic and meromorphic functions can be constructed using

\[ \zeta_0(w) \equiv \frac{\partial_{w} \theta_1(w | it)}{\theta_1(w | it)} + \frac{2\pi w}{t} \]

- Important \( \zeta_0 \) has non vanishing monodromy around cylinder

\[ \zeta_0(w + 1) = \zeta_0(w) + \frac{2\pi}{t} \]
Higher genus solutions

- $H$ and $L^A$ are given by

\[
H = -2 \text{Im} \sum_{n=1}^{N} c_n \zeta_0 (w - x_n) - 2 \text{Im} \sum_{n'=1}^{N'} c_{n'} \zeta_0 (w' + x'_{n'})
\]

\[
L^A = L^A_{\infty} + \sum_{p=1}^{P} l^A_p \zeta_0 (w - y_p) + \sum_{p'=1}^{P'} l^A_{p'} \zeta_0 (w' + y'_{p'})
\]

- Here $w' = \frac{it}{2} - w$ and hence $x_p, y_p$ are the location of poles on one boundary and $x'_{p'}, y'_{p'}$ on the other boundary.

- Boundary and regularity conditions can be solved using the properties of $\zeta_0$.

Regular is

\[
P = 2N, \quad P' = 2N'
\]
Higher genus solutions

- Holographic BCFT with only one asymptotic region $N = 1, P = 2$ and $N' = 0, P' = 0$.

- What happens to conservation of charge $Q^n_A \equiv \oint_{C \times S^2} dB^A$?. Nontrivial cycle on annulus.

- Monodromy of $\zeta_0$ leads to nontrivial monodromy of scalars as $w \to w + 1$. Compare monodromy of axion around a D7 brane.

- Conjecture: Such solutions correspond to the near horizon limit of a self dual string ending on a D7 brane wrapped on K3.
Discussion

- Calculation of observables like the entanglement entropy is (relatively) easy
- Calculation of correlation function (bulk/boundary in CFT)
- Generalization of solution to $\Sigma$ with an arbitrary number of boundaries and handles is straightforward.
- Is it possible to have solutions with no asymptotic AdS regions? (Cf Bachas talk last week).
- Do these solution correspond to string networks?