Ph 108 Midterm Solutions

TA: John Joseph M. Carrasco (jjmc@physics.ucla.edu)

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1. Consider a light pulse with the following parameters:

Pulse width=10 fs = 10^{-14} sec

Ceneter wavelength = 600 $\times 10^{-9}$ meter

Pulse energy = 1 millijoule

Shape is approximately rectangular

- **A. The pulse is propagating through vacuum**

  - **i. (5 pt) Caluclate the peak power in the pulse at the start**

$P = \frac{\text{Pulse Energy}}{\text{Pulse width}} = \frac{1\text{ mJ}}{10\text{ fs}} = 1 \times 10^{11}$ watts

  - **ii. (5 pt) Calculate the pulse width (in time domain) after it has gone 100 meters of vacuum**

Unchanged.

B. The pulse is propagating through normal air having its refractive index *n* given by:

$(n-1) = A\left(1 + \frac{B}{\lambda^2}\right) \Longrightarrow n = A + \frac{A B}{\lambda^2} + 1$

With $A = 28.71 \times 10^{-5}$, $B = 5.67 \times 10^{-3}$, and λ in nm

  - **i. (25 points) (10 theory, 15 numerics)**

Calculate the approximate pulse width (in time domain) after it has propagated the following distances through normal air:

Note : $\Delta f = \frac{1}{\textit{pulse width}} = 10^{14}$ *Hz,* $f_0 = c/600\,nm = 5 \times 10^{14}$ *Hz.*

$f_+ = 4.5 \times 10^{14}$, $f_- = 6.5 \times 10^{14} \Longrightarrow \lambda_+ = 667$ *nm,* $\lambda_- = 545$ *nm*

For $\lambda_+ = .667\ \mu$m, $n = 1.000290759018317$, $v_+ = c/n = 2.999127976493748 \times 10^8$ (not all significant digits)

For $\lambda_- = .545\ \mu$m, $n = 1.000292580538675$, $v_- = c/n = 2.999122515118974 \times 10^8$ (not all significant digits)

2 viable solutions

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1) If you assume the "slower" frequency enters the medium 10^-14 seconds after the faster frequency, (i.e. slower at the back of the pulse, faster at the front), we can find the new width of wave pulse by adding to the width the difference in propagation time between the faster and the slower pulses to reach the mark:

$w(d) = \left(w_0 + \frac{d}{v_-}\right) - \frac{d}{v_+}$

For 1/10 meters: $1.0607173 \times 10^{-14}$ seconds (not all significant digits)

For 1 meter: $1.6071735 \times 10^{-14}$ seconds

For 100 meter: $6.1717345 \times 10^{-13}$ seconds

2) You can calculate how long the center wavelength takes to reach the given distance. You can see how far each of the other frequencies propagate in that amount of time. Taking their difference gives the amount of spacial broadening to the pulse, which you can (approximately) convert to a temporal broadening by dividing by the speed of light in a vacuum, or equally valid to order of approximation the speed of the center frequency light in the medium. This broadening would simply be added to the original pulse width

$\Delta x(d) = \left|(v_+ - v_-) \frac{(d)}{v_0}\right| \Longrightarrow$

speed of light in vacuum: $\Delta w(d) = \frac{\Delta x(d)}{c}$

speed of center frequency in media: $\Delta w(d) = \frac{\Delta x(d)}{v_0} = \left|(v_+ - v_-) \frac{(d)}{v_0^2}\right|$

In either case the new width would be: $w_0 + \Delta w(d)$, and comparable to the numbers above.

- **scratch**

- **ii. (10 points: 7 "theory", 3 numerics) Peak power in pulse for each of the cases above**

$P = \frac{\text{Pulse Energy}}{\text{Pulse width}} = \frac{1\text{ mJ}}{\text{width}}$

- **scratch**

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2. (10 points, 7 theory + 3 numerics) An optical beam propagating through a medium has the following parameters:

$L = 10\,\text{cm}$

$I_{out} = 0.9\, I_{in}$

(theory: 7 points)

Determine I_{out} for L=20 cm

Determine I_{out} for L=100 cm

Determine I_{out} for $L = 1000$ cm

Solution

This system must follow Beer's Law: $I_{out} = e^{-\alpha l} I_{in}$, which at 10 centimeters,
has the drop off of .9, giving us $\alpha = 1.05361\, m^{-1}$, or .0105361 cm^{-1}

Determine I_{out} for L=20 cm

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Exp[-.2 1.0536051565782627`]
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0.81
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Determine I_{out} for L=100 cm

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Exp[-1 1.0536051565782627`]
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0.348678
```

Determine I_{out} for L = 1000 cm

```
Exp[-10 1.0536051565782627`]
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0.0000265614
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- **scratch**

3. Consider the following long thin long medium in which we have acvieved population inversion. The gain occurs ona naturally brodened transition.

Expression for gain is:

$$g(\nu) = \frac{g(0)}{1+\left[2\,\frac{(\nu-\nu 0)}{\Delta \mathrm{v}_N}\right]^2}$$

where g(0) is the gain at the center of the transition, ν_0 is the center frequency of the transition and $\Delta \mathrm{v}_N$ is the natural linewidth of the transition (FWHM). Spontaneous emission emanating from l≈0 has a linewid = $\Delta \mathrm{v}_N$(FWHM) and experiences exponential gain as it propagates along the axis of the amplifying medium.

- **A)**

(30 points for general idea, small dings for minor arithmetic error) Derrive the expression for the linewidth of the amplified emission that emerges at the far right of the amplifying medium.

Hint: $\frac{I_{out}(\nu)}{I_{in}(\nu)} = e^{(g(\nu)\cdot l)}$. Calculate ν at which $\frac{I_{out}(\nu)}{I_{in}(\nu)} = .5 \times \frac{I_{out}(0)}{I_{in}(0)}$

I'm guessing, ν at which $\frac{I_{out}(\nu)}{I_{in}(\nu)} = .5\,\frac{I_{out}(\nu_0)}{I_{in}(\nu_0)}$.

Plugging the above definition for g into the definition for $\frac{I_{out}}{I_{in}}$, and taking the natural logarithm I have:

$$g[\nu]\ l == \mathrm{Log}[1/2] + g_0\ l \Longrightarrow$$

$$\frac{l\, g_0}{1+\frac{4(\nu-\nu 0)^2}{\Delta\nu_N^2}} = -\mathrm{Log}[2] + l\, g_0 \Longrightarrow$$

$$\frac{l\,g_0}{-\text{Log}[2]+l\,g_0} == 1 + \frac{4\,(\nu-\nu0)^2}{\Delta\nu_N^2} \Longrightarrow$$

$$-1 + \frac{l\,g_0}{-\text{Log}[2]+l\,g_0} == \frac{4\,(\nu-\nu0)^2}{\Delta\nu_N^2} \Longrightarrow$$

$$\left(-1 + \frac{l\,g_0}{-\text{Log}[2]+l\,g_0}\right)\Delta\nu_N^2 == 4\,(\nu-\nu0)^2 \Longrightarrow$$

$$\nu0 \pm \frac{\Delta\nu_N}{2}\sqrt{\frac{\text{Log}[2]}{l\,g_0-\text{Log}[2]}} == \nu$$

Therefore linewidth is: $\Delta\nu_N \sqrt{\frac{\text{Log}[2]}{-\text{Log}[2]+l\,g_0}}$.

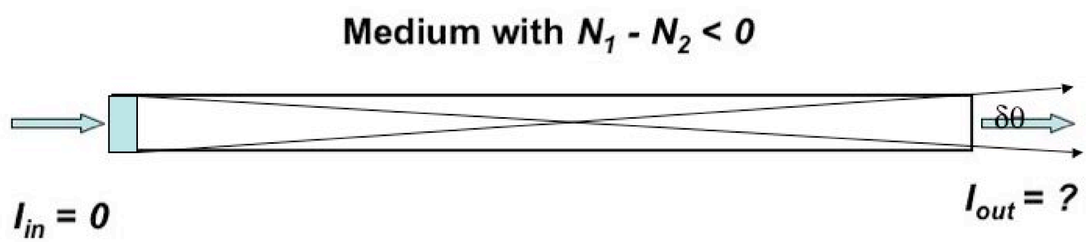
- **scratch**

- **B)**

For $\Delta v_N = 100$ MHz, g(0)=10% / cm, $l \simeq 10\,m$, and diam of the gain medium = 1 cm:

i. (10 points: 8 theory, 2 numerics)*Calculate the linewidth of the amplified spontaneous emission at l=10 meters.*
Plug into above formula or use approximation given in class ~ 11±3 MHz
(8.35 MHz w/ above formula, or 14.14 MHz as given in class)

ii.) (5 points for the good effort) Calculate the divergence at 10 m. I gave credit in general to variants of Divergence = $\frac{D_{f_{\text{inal}}}-D_{i_{\text{nitial}}}}{l_{\text{ength}}}$, both for people assuming a point source at the beginning of the medium, and measuring $D_f = 1$ cm, l=10 meter, as well as people who, following the graphic on the exam:



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calculated an expected beam-waist at the center, and looked at the difference between the beam-waist and the final diameter, over the distance $l = 5\,m$.

There was also the approach of just calculating the solid angle: $\sim \frac{\text{Area}}{l^2}$, which is the square of the above Divergence measure, which I also considered a valid measure of divergence.

- **scratch**