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Euclideanization, topological theories, higher dimensions and all that

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It is shown that the euclideanized Yukawa theory, with the Dirac fermion belonging to an irreducible representation of the Lorentz group, is not bounded from below. A one parameter family of supersymmetric actions is presented which continuously interpolates between the $N=2$ SSYM and the $N=2$ supersymmetric topological theory. In order to obtain a theory which is bounded from below and satisfies Osterwalder–Schrader positivity, the Dirac fermion should belong to a reducible representation of the Lorentz group and the scalar fields have to be reinterpreted as the extra components of a higher dimensional vector field.

1. Introduction

In earlier papers we presented a new method for euclideanization of Dirac [1], Majorana and Weyl [2] fermions. Unlike the usual procedure, the new procedure was continuous, the fermionic degrees of freedom were not doubled and the euclidean theory manifested all the symmetries of the original Minkowski space theory.

In this paper we discuss the euclideanization of theories with Yukawa couplings. It turns out, quite surprisingly, that the result of euclideanization depends crucially on the choice of fermionic representation of the Lorentz group.

If a Dirac fermion belonging to the four component irreducible representation is used, the euclidean action for a theory with pseudoscalar Yukawa couplings turns out to be bottomless. In particular we have found a one parameter family of actions which interpolate between the $N=2$ supersymmetric Yang–Mills theory [3] (SSYM) in Minkowski space and the euclidean supersymmetric topological theory [4] (SST) in flat space–time. We also show that this SST is not only hermitean but also satisfies the Osterwalder–Schrader (OS) positivity condition. However, this action does not lead to the correct euclidean Green functions.

On the other hand, if the fermion belongs to a reducible, eight component representation of the Lorentz group, the euclideanized Yukawa action *is* bounded from below and manifests all the symmetries of the original Minkowski space theory. Such a theory also yields the correct euclidean Green functions. The requirement of OS positivity of such a euclidean theory demands that the scalar fields be viewed as the fourth and the fifth components of a six dimensional vector field.

Euclideanization of theories with Yukawa couplings and the fermions in the irreducible representation of the Lorentz group is discussed in section 2. Euclideanization using a reducible representation is discussed in section 3 and the OS positivity of the euclidean theory is discussed in section 4. Finally the conclusions and discussion are given in section 5.

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2. $N=2$ SST

Consider the $N=2$ SSYM, in four dimensional Minkowski space, with complex supersymmetry, i.e. Dirac fermions [3]:

$$S=\int d^4x\,\{\tfrac{1}{2}[D_\mu S\,D^\mu S+D_\mu P\,D^\mu P-(P\times S)^2]-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+i\bar\psi(\not{D}-S+i\gamma_5 P)\psi\}\,. \tag{1}$$

(All the gauge group indices are suppressed for notational convenience.) Here $(P\times S)^a=f^{abc}P^bS^c$, S and P are the scalar and the pseudoscalar fields respectively and ψ is the Dirac fermion transforming according to the usual irreducible representation of the Lorentz group. The notation is as in ref. [1].

This action is invariant under the chiral transformations

$$\delta_\alpha\psi=\tfrac{1}{2}i\gamma_5\alpha\psi,\quad \delta_\alpha S=-\alpha P,\quad \delta_\alpha P=\alpha S\,, \tag{2}$$

and the supersymmetry transformations

$$\delta_\epsilon A_\mu=i(\bar\epsilon\gamma_\mu\psi-\bar\psi\gamma_\mu\epsilon),\quad \delta_\epsilon S=i(\bar\epsilon\psi-\bar\psi\epsilon),\quad \delta_\epsilon P=\bar\epsilon\gamma_5\psi-\bar\psi\gamma_5\epsilon\,,$$
$$\delta_\epsilon\psi=[\Sigma_{\mu\nu}F^{\mu\nu}+\not{D}(i\gamma_5P-S)-i\gamma_5P\times S]\epsilon\,. \tag{3}$$

We now continue this theory to euclidean space. As discussed in ref. [2], the euclideanization procedure should be such that the interpolating theory, and hence the euclidean theory, manifests all the symmetries of the original Minkowski space theory and the euclidean theory satisfies the OS positivity condition. Further, it was also argued that, to satisfy these requirements, one is free to insert arbitrary powers of the interpolating metric in the interpolating action.

It is easy to verify that the interpolating action

$$S^\theta=\int d^4x\sqrt{-g^\theta}\,(\tfrac{1}{2}\{g^\theta_{\mu\nu}D^\mu S\,D^\nu S+(-g^\theta)^{-1}[g^\theta_{\mu\nu}D^\mu P\,D^\nu P-(P\times S)^2]\}$$
$$-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+i\psi^\dagger M[(g^\theta)^{\mu\nu}\gamma^\theta_\mu D_\nu-S+i(-g^\theta)^{-1}\gamma^\theta_5P]\psi) \tag{4}$$

is invariant under the interpolating Lorentz transformations [1] for all the allowed values of the complex interpolation parameter θ. Here, $M=\gamma_0$, $g^\theta_{\mu\nu}$ and γ^θ_μ are the interpolating metric and gamma matrices and θ is the complex interpolation parameter, $0\leqslant\theta\leqslant\frac{1}{2}\pi$, $\theta\neq\frac{1}{4}\pi$ (see ref. [1] for details). This action is invariant under the chiral transformations

$$\delta_\alpha\psi=\tfrac{1}{2}i\gamma^\theta_5\alpha\psi,\quad \delta_\alpha S=-\alpha P,\quad \delta_\alpha P=(-g^\theta)\alpha S\,. \tag{5}$$

The factors of g^θ are necessary because $(\gamma^\theta_5)=-g^\theta$. Without these factors in the action and the chiral transformations, we will not have chiral invariance for any value of $\theta\neq0$. The supersymmetry transformations which leave (4) invariant are

$$\delta_\epsilon A_\mu=i(\epsilon^\dagger M\gamma^\theta_\mu\psi-\psi^\dagger M\gamma^\theta_\mu\epsilon),\quad \delta_\epsilon S=i(\epsilon^\dagger M\psi-\psi^\dagger M\epsilon),\quad \delta_\epsilon P=\epsilon^\dagger M\gamma^\theta_5\psi-\psi^\dagger M\gamma^\theta_5\epsilon\,,$$
$$\delta_\epsilon\psi=\{\Sigma^\theta_{\mu\nu}F^{\mu\nu}+\not{D}[(-g^\theta)^{-1}i\gamma^\theta_5P-S]-(-g^\theta)^{-1}i\gamma^\theta_5P\times S\}\epsilon\,. \tag{6}$$

The interpolating action is also invariant under parity, charge conjugation and time reversal. The transformation properties of fermions and vector fields under these is as given in ref. [1] and the scalar fields transform as

$$\mathscr{C},\mathscr{T}:\ S=-S,\quad \mathscr{P}:\ S=S,\quad \mathscr{T}:\ P=P,\quad \mathscr{P},\mathscr{C}:\ P=-P\,. \tag{7}$$

Thus we have a one complex parameter family of $N=2$ SSYM actions which have all the symmetries of the original Minkowski space action. When the parameter $\theta=0$, the action (4) and the transformations (5) and

(6) reduce to the Minkowski space action (1) and transformations (2) and (3) respectively. When the interpolating parameter $\theta=\frac{1}{2}\pi$ we get the following *hermitian* euclidean action (the euclidean metric is $g^{\mathrm{E}}_{\mu\nu}=-\delta_{\mu\nu}$ and $(\gamma^{\mathrm{E}}_{\mu})^{\dagger}=-\gamma^{\mathrm{E}}_{\mu}$, $(\gamma^{\mathrm{E}}_{5})^{\dagger}=-\gamma^{\mathrm{E}}_{5}$, $M=\mathrm{i}\gamma^{\mathrm{E}}_{5}$):

$$S^{\mathrm{E}}=\int \mathrm{d}^4x\,\{\tfrac{1}{2}[\mathrm{D}_\mu S\,\mathrm{D}^\mu S-\mathrm{D}_\mu P\,\mathrm{D}^\mu P+(P\times S)^2]-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\mathrm{i}\psi^{\dagger}\,\mathrm{i}\gamma^{\mathrm{E}}_5(\not{D}^{\mathrm{E}}-S-\mathrm{i}\gamma^{\mathrm{E}}_5 P)\psi\}\,, \tag{8}$$

which is invariant under the following chiral *scale* transformations:

$$\delta_\alpha\psi=\tfrac{1}{2}\mathrm{i}\gamma^{\mathrm{E}}_5\alpha\psi,\quad \delta_\alpha S=-\alpha P,\quad \delta_\alpha P=-\alpha S\,, \tag{9}$$

and the supersymmetry transformations

$$\delta_\epsilon S=\mathrm{i}(\epsilon^{\dagger}\mathrm{i}\gamma^{\mathrm{E}}_5\psi-\psi^{\dagger}\mathrm{i}\gamma^{\mathrm{E}}_5\epsilon),\quad \delta_\epsilon P=\epsilon^{\dagger}\mathrm{i}\gamma^{\mathrm{E}}_5\gamma^{\mathrm{E}}_5\psi-\psi^{\dagger}\mathrm{i}\gamma^{\mathrm{E}}_5\gamma^{\mathrm{E}}_5\epsilon,\quad \delta_\epsilon A_\mu=\mathrm{i}(\epsilon^{\dagger}\mathrm{i}\gamma^{\mathrm{E}}_5\gamma^{\mathrm{E}}_\mu\psi-\psi^{\dagger}\mathrm{i}\gamma^{\mathrm{E}}_5\gamma^{\mathrm{E}}_\mu\epsilon)\,,$$

$$\delta_\epsilon\psi=[\Sigma^{\mathrm{E}}_{\mu\nu}F^{\mu\nu}-\not{D}(S+\mathrm{i}\gamma^{\mathrm{E}}_5P)+\mathrm{i}\gamma^{\mathrm{E}}_5P\times S]\epsilon\,. \tag{10}$$

Eqs. (8)–(10) can be put in the standard form [4] of the $N=2$ SST by redefining the euclidean metric and gamma matrices as $g_{\mu\nu}=\delta_{\mu\nu}$, $\gamma_\mu=\mathrm{i}\gamma^{\mathrm{E}}_5\gamma^{\mathrm{E}}_\mu=\gamma^{\dagger}_\mu$ and $\gamma_5=\mathrm{i}\gamma^{\mathrm{E}}_5=\gamma^{\dagger}_5$. In this notation the scalar and the pseudoscalar fields S and P appear to have pseudoscalar and scalar couplings respectively.

Since the interpolating action, and hence the euclidean action, is invariant under $\mathscr{C}$, $\mathscr{P}$ and $\mathscr{T}$, the euclidean action is also invariant under the OS reflection operator $\Theta=\mathscr{C}\mathscr{T}$. From (7) we see that $\Theta S=S$ and $\Theta P=-P$. The OS positivity of the euclidean theory (8) can now be shown using a procedure similar to that of ref. [1]. In the usual formulation of the SST, S and P are taken to be pseudoscalar and scalar fields respectively, i.e. $\Theta S=-S$ and $\Theta P=P$, leading to the lack of OS positivity.

The euclidean Green functions can now be obtained [1,2] by performing the functional differentiation of the partition function obtained from the above action. Although the above euclideanization procedure satisfies all the symmetry requirements [2], the euclidean theory does not yield the correct euclidean Green functions. For example, the propagator for the pseudoscalar field P, obtained from the above euclidean theory, does not agree with the propagator obtained by directly continuing the Minkowski space propagator.

Hence a different procedure is needed which yields the "correct" euclidean Green functions. This, as we will now show, can be achieved by using fermions belonging to a *reducible* representation of the Lorentz group.

3. The six dimensional formulation

It has been shown by Brink et al. [3] that the $N=2$ SSYM can be obtained by a trivial dimensional reduction of a six dimensional $N=1$ SSYM:

$$S^{\mathrm{M}}=\int \mathrm{d}^4x\,[-\tfrac{1}{4}F_{Q,R}F^{Q,R}+\mathrm{i}(\bar{\lambda}P_-)\,\hat{\not{D}}(P_+\lambda)]\,. \tag{11}$$

Here the fields are functions of only the usual four space-time dimensions and the other notation is as follows:

$$Q,R=0,\ldots,5,\quad \mu=0,\ldots,3,\quad g_{QR}=(1,-1,\ldots,-1),\quad \{\Gamma_Q,\Gamma_R\}=2g_{QR}\,,$$

$$\Gamma_\mu=\sigma_1\otimes\gamma_\mu,\quad \Gamma_4=\mathrm{i}\sigma_2\otimes I_4,\quad \Gamma_5=\mathrm{i}\sigma_1\otimes\gamma_5,\quad \Gamma_7=\sigma_3\otimes I_4,\quad P_\pm=\tfrac{1}{2}(1\pm\Gamma_7)\,,$$

$$\lambda=\begin{pmatrix}\psi\\ \phi\end{pmatrix},\quad \xi=\begin{pmatrix}\epsilon\\ \eta\end{pmatrix},\quad \hat{\not{D}}=\Gamma_\mu(\partial^\mu+A^\mu)+\Gamma_4A^4+\Gamma_5A^5,\quad A^4=S,\quad A^5=P\,. \tag{12}$$

The six dimensional Dirac spinor λ has eight components. In the above choice of gamma matrices the upper and the lower four components of λ correspond to four dimensional Dirac spinors ψ and ϕ. Similarly for the SUSY parameter ξ. The six dimensional chirality projection operator P_+ picks out the upper four components. Thus the action (11) contains a Dirac fermion in a reducible representation of SO(3, 1). It is easy to verify, by direct substitution, that (11) is equal to (1).

Action (11) is invariant under the chiral phase transformations (which, in this notation, are rotations in the (4, 5) plane)

$$\delta_\alpha\lambda=\tfrac{1}{2}\Gamma_4\Gamma_5\lambda\alpha,\quad \delta_\alpha A_4=-A_5\alpha,\quad \delta_\alpha A_5=+A_4\alpha\,. \tag{13}$$

and supersymmetry transformations

$$\delta_\epsilon A_Q=\mathrm{i}(\bar{\xi}\Gamma_Q P_+\lambda-\bar{\lambda}\Gamma_Q P_+\xi),\quad \delta_\epsilon\lambda=\Sigma_{QR}F^{QR}\xi\,. \tag{14}$$

The action (11) for six dimensional Weyl fermions can be euclideanized by following a procedure similar to that of ref. [2] for four dimensional Weyl fermions. To this end, we defined the interpolating metric and gamma matrices as

$$g^\theta_{QR}=\left(\frac{\cos 2\theta}{|\cos 2\theta|},-1,\ldots,-1\right),\quad 0\leqslant\theta\leqslant\tfrac{1}{2}\pi,\quad \theta\neq\tfrac{1}{4}\pi\,,$$

$$\Gamma^\theta_0=\frac{1}{\sqrt{|\cos 2\theta|}}(\Gamma_0\cos\theta+\mathrm{i}\Gamma_7\sin\theta),\quad \Gamma^\theta_7=\frac{1}{\sqrt{|\cos 2\theta|}}(\Gamma_7\cos\theta-\mathrm{i}\Gamma_0\sin\theta)\,,$$

$$M=\Gamma_0,\quad P_\pm=\frac{1}{2}\left(1\pm\frac{\Gamma^\theta_7}{\sqrt{-g^\theta}}\right) \tag{15}$$

with all the other gamma matrices independent of θ. The interpolating $N=2$ SSYM is defined as

$$S^\theta=\int \mathrm{d}^4x\sqrt{-g^\theta}\,[-\tfrac{1}{4}F_{Q,R}F^{Q,R}+\mathrm{i}(\lambda^\dagger MP_-)\,\hat{\not{D}}^\theta(P_+\lambda)]\,. \tag{16}$$

Since Γ_4 and Γ_5 are independent of θ – the chiral transformations too are the same as in (13) for all θ – i.e. chiral *phase* transformations. It is easy to show that the action (16) is invariant under chiral and supersymmetry transformations similar to (13) and (14), with Γ_R and Γ_7 replaced by Γ^θ_R and Γ^θ_7 respectively.

For $\theta=\frac{1}{2}\pi$ the action (16) yields a euclidean $N=2$ SSYM. We choose the euclidean gamma matrices such that λ still separates into two Dirac fermions as in (12). Such a choice is $\Gamma^{\mathrm{E}}_0=\mathrm{i}\sigma_1\otimes\gamma_0$, $\Gamma^{\mathrm{E}}_7=\mathrm{i}\sigma_3\otimes I_4$ and other gamma matrices as in (12). Thus the euclidean action obtained from (16)

$$S^{\mathrm{E}}=\int \mathrm{d}^4x\,[-\tfrac{1}{4}F_{Q,R}F^{Q,R}-\mathrm{i}(\lambda^\dagger P_-)\,\hat{\not{D}}^{\mathrm{E}}(P_+\lambda)]\,, \tag{17}$$

can be written in the four component notation as

$$S^{\mathrm{E}}=\int \mathrm{d}^4x\,\{\tfrac{1}{2}[\mathrm{D}_\mu S\,\mathrm{D}^\mu S+\mathrm{D}_\mu P\,\mathrm{D}^\mu P-(P\times S)^2]-\tfrac{1}{4}F^a_{\mu\nu}F^{\mu\nu a}-\mathrm{i}\phi^\dagger(\not{D}^{\mathrm{E}}-S+\mathrm{i}\gamma_5 P)\psi\}\,. \tag{18}$$

Here, $g^{\mathrm{E}}_{\mu\nu}=-\delta_{\mu\nu}$, $\not{D}^{\mathrm{E}}=\gamma^{\mathrm{E}}_\mu\mathrm{D}^\mu$, $\gamma^{\mathrm{E}}_\mu=-(\gamma^{\mathrm{E}}_\mu)^\dagger$ and $\gamma^\dagger_5=\gamma_5$. This action is invariant under the supersymmetry transformations

$$\delta_\epsilon A_\mu=-\mathrm{i}(\eta^\dagger\gamma^{\mathrm{E}}_\mu\psi-\phi^\dagger\gamma^{\mathrm{E}}_\mu\epsilon),\quad \delta_\epsilon S=-\mathrm{i}(\eta^\dagger\psi-\phi^\dagger\epsilon)\,,\quad \delta_\epsilon P=-(\eta^\dagger\gamma_5\psi-\phi^\dagger\gamma_5\epsilon)\,,$$

$$\delta_\epsilon\psi=[\Sigma_{\mu\nu}F^{\mu\nu}+\not{D}^{\mathrm{E}}(\mathrm{i}\gamma_5 P-S)-\mathrm{i}\gamma_5 P\times S]\epsilon\,, \tag{19}$$

where we have substituted for λ and ξ from (12). The actions (16) and (18) are not real and the transformations (19) are not unitary. This difficulty can be avoided, as in ref. [2], by redefining the hermitean conjugation operator $\mathscr{H}$, $\forall\theta$, as $\mathscr{H}A=A^\dagger$ for any complex matrix A and $\mathscr{H}(P_\pm\lambda)=\lambda^\dagger MP_\mp M$. For $\theta=0$ $\mathscr{H}$ reduces to the usual hermitian conjugation. Although the euclidean action involves two different Dirac fermions ψ and ϕ there is no doubling of fermionic degrees of freedom. This is because, as was argued in ref. [2], the fermion $P_+\lambda$

satisfies the six dimensional Weyl condition for *all* the values of θ. Since the two Dirac fermions are independent it is possible to have the euclidean chiral transformations to be chiral phase transformations and hence have an action bounded from below.

Thus we have continued the $N=2$ SSYM to a euclidean $N=2$ SSYM such that the euclidean action is bounded from below and has all the symmetries of the original Minkowski space theory. It can be easily verified that the euclidean action (19) also leads to the "correct" euclidean propagators. We now discuss the OS positivity of this euclidean theory.

4. OS positivity

The interpolating action for a four dimensional free Dirac fermion, written in six dimensional notation, is obtained from (16) by setting the vector fields to zero. This action is invariant under the discrete transformations $\mathscr{C}$, $\mathscr{P}$ and $\mathscr{T}$ with the transformation properties of the fermion λ given by

$$\mathscr{C}\lambda=\Gamma_1\Gamma_3\Gamma_5(\bar{\lambda})^{\mathrm{T}}, \quad \mathscr{P}\lambda=\mathrm{i}\Gamma_1\Gamma_2\Gamma_3\Gamma_5\lambda, \quad \forall\theta, \tag{20}$$

$$\mathscr{T}\lambda=\mathrm{i}\Gamma_0\Gamma_2\Gamma_4\Gamma_5\lambda, \quad \forall\theta<\tfrac{1}{4}\pi, \qquad \mathscr{T}\lambda=\mathrm{i}\Gamma_0\Gamma_2\Gamma_4\lambda, \quad \forall\theta>\tfrac{1}{4}\pi. \tag{21}$$

Using these and the transformation properties of the scalar fields (7) it can be shown that the interpolating action (16) is invariant under $\mathscr{C}$, $\mathscr{P}$ and $\mathscr{T}$ for $\theta<\frac{1}{4}\pi$. However, for $\theta>\frac{1}{4}\pi$ the action (16) is invariant under $\mathscr{C}$ and $\mathscr{P}$ but not under $\mathscr{T}$. Hence the OS reflection operator $\Theta=\mathscr{C}\mathscr{T}$ does not leave the action invariant.

To overcome this difficulty and to define a euclidean theory with OS positivity, we change the definition of time reversal for $\theta>\frac{1}{4}\pi$ as follows. Let us take the higher dimensional formulation serious and actually treat the scalars as the extra components of a six dimensional vector field.

We denote the usual four dimensional discrete transformations by $\mathscr{C}$, $\mathscr{P}$ and $\mathscr{T}$, and reflection around the fourth and the fifth space dimension by $\mathscr{P}_4$ and $\mathscr{P}_5$. In Minkowski space the scalar fields transform under the discrete transformations as given by (7). We choose the discrete transformations of A_4 and A_5 to be the same as those of the scalar fields. The operators which generate such transformations on A_4 and A_5 are obtained as follows:

$$\mathscr{P}S=S, \quad \mathscr{P}P=-P \Rightarrow \mathscr{P}\mathscr{P}_5A_4=A_4, \quad \mathscr{P}\mathscr{P}_5A_5=-A_5,$$

$$\mathscr{C}S=-S, \quad \mathscr{C}P=-P \Rightarrow \mathscr{C}\mathscr{P}_4\mathscr{P}_5A_4=-A_4, \quad \mathscr{C}\mathscr{P}_4\mathscr{P}_5A_5=-A_5,$$

$$\mathscr{T}S=-S, \quad \mathscr{T}P=P \Rightarrow \mathscr{T}\mathscr{P}_4A_4=-A_4, \quad \mathscr{T}\mathscr{P}_4A_5=A_5. \tag{22}$$

Thus the six dimensional theory is invariant under $\mathscr{P}\mathscr{P}_5$, $\mathscr{C}\mathscr{P}_4\mathscr{P}_5$ and $\mathscr{T}\mathscr{P}_4$, and hence under $\mathscr{C}\mathscr{P}\mathscr{T}\mathscr{P}_4^2\mathscr{P}_5^2=\mathscr{C}\mathscr{P}\mathscr{T}$ for $\theta<\frac{1}{4}\pi$. For $\theta>\frac{1}{4}\pi$ the OS reflection operator on S and P is $\Theta=\mathscr{C}\mathscr{T}$. The corresponding OS reflection operator for A_4 and A_5, from (22), is $\mathscr{C}\mathscr{T}\mathscr{P}_5$. This, however, is not the correct OS reflection operator in six dimensions, due to the extra $\mathscr{P}_5$. To correct for this we redefine the six dimensional time reversal operator for $\theta>\frac{1}{4}\pi$ to be $\mathscr{T}\mathscr{P}_4\mathscr{P}_5$. Along with the six dimensional charge conjugation operator as in (22) this new time reversal leads to the correct OS reflection operator in six dimensions, namely, $\Theta=\mathscr{C}\mathscr{T}$. From (22) we see that under this new time reversal

$$\mathscr{T}\mathscr{P}_4\mathscr{P}_5A_4=-A_4, \quad \mathscr{T}\mathscr{P}_4\mathscr{P}_5A_5=-A_5, \quad \forall\theta>\tfrac{1}{4}\pi. \tag{23}$$

The euclidean theory is indeed invariant under this time reversal and hence, along with charge conjugation as defined in (22), satisfies the OS positivity too.

Notice that this redefinition of euclidean time reversal also agrees with the difference in the definitions (21) of time reversal of λ for $\theta<\frac{1}{4}\pi$ and $\theta>\frac{1}{4}\pi$. These definitions differ by a factor of Γ_5 which is precisely the factor generated by a space reflection around the fifth space dimension.

Thus the euclideanized $N=2$ SSYM, obtained using fermions belonging to a reducible representation of the Lorentz group, satisfies the OS positivity *only* if the scalar fields are treated as the extra components of a six dimensional vector field.

It is worth pointing out that the euclidean action (18) looks similar to that obtained using the usual continuation procedure. Hence the problem of the OS positivity of euclideanized Yukawa theory will arise even in that case. However, unlike our continuation procedure, with the usual continuation procedure there is no natural reason to change the definitions of discrete transformations of the scalar fields so as to get the OS positivity.

5. Conclusions and discussion

We have shown that there are two distinct ways of euclideanizing the $N=2$ SSYM such that the euclidean theory has all the symmetries of the Minkowski space theory.

If a Dirac fermion belongs to the usual, four component, irreducible representation of the Lorentz group, the euclidean chiral symmetry is chiral *scale* symmetry and the euclidean action is that of the $N=2$ supersymmetric topological theory. Furthermore, in euclidean space there is an apparent interchange of roles of the scalar and the pseudoscalar fields. A correct choice of discrete transformation properties of the scalar fields, valid for all θ, shows that the SST is *indeed* OS positive. However, the SST does not lead to the correct euclidean Green functions.

Hence a different euclideanization procedure involving fermions belonging to a reducible, eight component representation was developed. This six dimensional formulation led to a euclidean $N=2$ SSYM which was bounded from below and had all the symmetries of the original Minkowski space theory. In euclidean space the chiral transformations are *phase* transformations. The requirement of OS positivity demanded that the four dimensional scalar and pseudoscalar fields be viewed as the fourth and the fifth components of a six dimensional vector field.

The procedure set up for the $N=2$ SSYM is quite general and applies equally well to any theory involving fermions with Yukawa coupling – in particular to the Glashow–Salam–Weinberg model and the $N=4$ SSYM. The details of euclideanization of Majorana and Weyl fermions with Yukawa couplings is presented elsewhere [5]. The different euclideanization procedures also lead to a new understanding of chiral anomalies [5].

The work presented here raises many questions, some of which are listed here. Is there any fundamental reason to choose the reducible, rather than the irreducible, representation of SO(3, 1) for the fermion being euclideanized? What is the physical significance of the scalar fields being treated as the extra components of a higher dimensional vector field? Are there different ways of euclideanizing gravity too, such that, unlike the usual procedure, one of the ways leads to a euclidean gravity which is bounded from below?

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