Vlasov Eulerian Simulations in 2 Dimensions
Destruction of BGK type Equilibrium by a Magnetic Field

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It was shown that a two dimensional plasma can reach an equilibrium of BGK Type. The effect of a Magnetic Field on such a Plasma is to destroy the phase space holes generated by the initial state.

1. The code

An Eulerian code based on a time splitting scheme and a flux balance method [1] to solve the so splitted equations, is used to solve the 2D, periodic in space, Vlasov equation (1).

$$\frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} + v_y \frac{\partial f}{\partial y} + (E_x + v_y \omega_c) \frac{\partial f}{\partial v_x} + (E_y + v_x \omega_c) \frac{\partial f}{\partial v_y}(x, y, v_x, v_y, t) = 0 \qquad (1)$$

ω_c is the gyrofrequency in plasma frequency (ω_p) units. The time t is normalized to ω_p^{-1} , the space variables to Debye length, the velocities to thermal velocity (v_{th}). The electric field is computed resolving Poisson equation by Fourier transform.

2. The Equilibrium State

Starting the time evolution computation with a two stream initial condition (2).

$$F(x,y,v_x,v_y,t=0) = \exp(-\{(v_x-2)^2+(v_y-2)^2\}/2)\ (1+5(v_x^2+v_y^2))(1+pert)/12\ \pi \qquad (2)$$

with pert being a cosine term in x and in y, the plasma evolve to an equilibrium function of energy, i.e., a BGK equilibrium

$$f(x,y,v_x,v_y,t) = f((v_x^2+v_y^2)/2 + \phi(x,y)) \qquad (3)$$

with $\phi(x,y)$ the electric potential.

With the actual code and initial condition brought as example here, that equilibrium is obtained at T=40 ω_p^{-1}.

3. The Magnetic Field Influence

At T>44 ω_p^{-1} a magnetic field is turned on. The figures are for a gyrofrequency ω_c=1.01 ω_p. After a few time steps (the time step in the actual simulation is Δt : .25 ω_p^{-1}) the phase space holes structures disappear, the global behaviour of the plasma in velocity space being of Maxwellian type, and only the high energy particles retain remembrance of the initial state (see figure 1).

The density (figure 2) and the electric field (figure 3) which were structured before the magnetic field is turn on (figures 2 and 3, t = 44 ω_p^{-1}) became totally chaotic after the magnetic field act for a long enough time (figures 2,3, T = 92 ω_p^{-1}).

When a strong magnetic field is acting ($\omega_c >> \omega_p$) a kind of structure appears in real space (Density). Further work must be done to be sure that structure is a direct influence of the magnetic field, and is not an effect due to the periodicity in space.

A very small magnetic field ($\omega_{c=}$ .1 ω_p^-) is enough to destroy the phase space structures and in this case between an attempt for holes creation, and annihilation effect due to the magnetic field. Again after a longer time the structure in density disappears.

Reference :

E. Fijalkow, Numerical Solution to Vlasov Equation : the 2D code, submitted to Computer Physics Communications.

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