Modeling of Coulomb's particles collisions in a plasma

S. A. Maiorov

Institute of General Physics of Russian Academy of Science

1. Introduction

The computer modeling by particles (ab initio) permit to look at modern kinetic theories in a new fashion. The fundamental properties of classical Coulomb's plasma was studied by many-particle dynamics method (reference and review see in [1]). The thermodynamic characteristics, microfield distributions, shielding, collisions, three-body recombination was studied. In this paper the problem of collisions in ideal full ionized Coulomb plasma is examined The modern theory of Coulomb's collisions in a plasma is based on the Landau work[2]. The Boltzmann collision integral can be simplified by the decomposition on a small energy of particles interaction. Divergence of the Landau integral eliminated by the restriction of the integration limits. Divergence because of the lower integration limit was caused by the assumption of a small moment transmit in a single collision. It was eliminated by the minimum impact parameter introduction. The cross sections cuting was motivated for far flights by: 1) the shielding effect in a plasma; 2) weak logarithmic dependence of the collision frequency from a cuting radius. First argument is based on the shielding effect for unmovable charge in a plasma. But the application of this effect to the dynamic shielding is not justified. The second argument does not valid for qualitative distinctions in choice of the upper integration limit. Doubtlessly, Debye radius is a top estimation for an upper integration limit.

The Balesku and Lennard approach is based on the concept of the dynamical chargeability in a plasma. But the significance of the dielectric capasitivity for a small wave numbers corresponds to cut off interaction on the Debye radius. Thus an effective radius of interaction is entered. N.N.Bogolubov has note, that there is an internal contradiction in the Boltzmann equation. A stochastic sequence assumption (the hypothesis of chaos - Stosszahlansatz) is done, at which a movement of molecules is treated as a random process and a binary collisions are considered. On the other hand, the cross sections are received from a dynamics equations. Koga [3] developed the approach to eliminating of divergence of Boltzmann collision integral in a plasma, which was based on accounting of the interaction with nearest neighbours. His explanation of the Boltzmann collision integral divergence is convincing.

The usial opinion is that a shielding is a physical reason of the Boltzmann collisions integral convergence. The question appears: what is the reason of the Boltzmann collisions integral convergence in a structures without shielding, for example, compensated semiconductors? It is clearly, that a free path is not equal to zero in a system of particles without shielding since a mean field for a located in space bodies is bounded value (Holtzmark result). The numerical computation on a Coulomb collisions in a plasma is important for clearing up of the plasma kinetic theories. Particularly, we can speak about greater significance of a interionic distance on the basis of the computer experiments results. Underestimation of this characteristic length traditionally comes from the gas kinetic theories. A mean distance between particles does not play any role in the gas kinetic theory - only a size of atom and a free path length matters. The interionic distance defines the characteristic size of the electric field discontinuity in a plasma. The Debye shielding radius substitutes frequently the interpartial distance in a plasma.

The Vlasov approach permits to avoid the divergence problem, but the Vlasov equation deals with the mean charges density. Vlasov marked [4], that for a plasma the account consecutive binary collisions does not reflect a reality, because it is impossible to satisfy the binary collisions conditions and it is necessary to take into account a collisions with impact parameter less of the half interionic distance. A Coulomb collisions in a plasma were studed by Persico [5], Lagmuire [6], Landau [2], Davidov [7], Druveystein [8], Spitzer [9-10] and others. According to the majority the top impact parameter should be set equal to the Debye radius. An interpartical distance as a cut-off impact parameter chosen Davidov, Cowling [11] and Chandrasecar [12], which studed a stellar dynamics, where the Debye shielding effect is absent.

Chandrasecar considered, that a divergence arises because of the wrong application of the two bodies problem decision for an account of a far flights and the cut-off impact parameter should be equal to a some more interpartcial distance.

At present, according to Landau, Spitzer and others, the cut-off impact parameter value should be equal to the Debye radius. The opinions are concentrated on the important problem to take into account a multiple collisions in a plasma by the binary Coulomb collision approximation. But while there is no a kinetic equations system for a satisfactory describing of a plasma dynamics in all cases, a various approaches are a methods for a particular problem decision, not possessing generality [3]. The modeling of a classical Coulomb plasma by the many - particle dynamics method permits to solve the problem of a mean free path length of electrons. A thermodynamic characteristics in an equilibrium plasma are depended on a non-ideality parameter of plasma. Therefore it is possible to detect a phenomenon value of the Coulomb factor (logarithm), depending on the non-ideality parameter. The many particles dynamics results are presented for the electron beam braking by ions problem.

2. Statement of the computer simulation using particles

The technique used here for research of a classical Coulomb plasma is described in [1,13]. We consider the time-evolution of a system from $n(1+z)$ particles, confined within a cubic volume with impenetrable walls. The trajectories of n positively charged particles (ions) and nz negatively charged particles (electrons) are calculated by solving the Newton equations:

$$d^2 r_k / dt^2 = F_k / m_k, \qquad F_k = \sum_{l \neq k}^{2n} f_{kl}, \qquad k = 1, 2, \ldots, 2n$$

Here $r_k(t)$- -is the k-th particle radius vector, m_k is its mass, q_k is its charge. The Coulomb force of the interaction f_{kl} between particles at distances between particles less r_0 was modernized as interaction mutually penetrable r_0 diameter spheres [13]. The r_0 value must be chosen sufficiently small in order not to influence on the calculation results. The validity of this condition was verified in study by r_0 variation from code run to run. At the initial moment, $t=0$, the coordinates of all the particles were assigned by the pseudorandom number generator corresponding to a uniform distribution in cube. The choice of the cube edge size a ensured the necessary density of a plasma. The initial velocities of electrons are corresponded uniform flow.

Particle - particle method [13] is most suitable method for numerical integration of the Newton equations at study of a plasma, in which are taken into account interaction of each particle with each. At realization of this method algorithm, the using specific character of classical Coulomb plasma for significant reduction of number of arithmetic operations was developed. In its basis is determination at each particle of the nearest particles and account of a interaction between them with more precision. At present, the method is advanced - for each particle four nearest particles was determined , the approximation of average force by linear function of a time is used, the control of an accuracy at each external step and automatic choice of a internal step, the prediction stage for calculation of average forces are entered. These improvements have allowed in some times to increase an accuracy and to increase number of taken into account particles. The code PLASMIC has the extensive diagnostics block. It includes calculation of athermodynamic characteristics, the distribution functions on kinetic, potential and full energy of electrons, ions and pairs of particles, on velocity projections, distributions of electric fields and them correlation functions, calculation kinetic coefficients, visualization of a separate particles trajectories is possible.

The technique used here differs from used earlier for determination of average free path [1]. The main purpose of the previous researches was study non ideal plasma. At present main attention is given to an ideal plasma. In this work were used periodic boundary conditions. The reason of failure from mirror walls consists in following. Close by mirror reflecting walls the effective density of plasma is in two times below. Particles feel smaller sum effect from other particles. The weaken of microfield for particles close by walls is not enough discounted at the previous computing experiments interpretation. The field distribution for thirty particles satisfactorily coincides with Holtzmark distribution. But this distribution is received for test particle in a center of sphere. Therefore, the mirror walls influence is important for collisions modeling. The second difference is called by the next specific character of a considered problem. The electron beam, moving parallel to a cube axes in periodic system, can accumulate disturbing effect, since they also appear periodic. To avoidance of this channeling effect, the beam has any direction so, that direction cosines did not make divisible numbers.

We shall consider the problem, from a modeling of which it is possible to determine the Coulomb logarithm. On a stationary charge ze from an infinity move with the velocity u u a flat flow of the particles with mass m, charge e, density of particles n. In the binary collisions approximation only interaction of particles with a stationary center is taken into account. Average force for the Coulomb center by the particles, possessing impact parameter less ρ_{max}, is equal

$$F = \Lambda \frac{4\pi z^2 e^4 n}{mu^2},$$

Where - $\Lambda = \ln \frac{\sqrt{\rho_{max}^2 + \rho_{min}^2}}{\rho_{min}} \approx \ln \frac{\rho_{max}}{\rho_{min}}$ Coulomb logarithm, $\rho_{min} = Ze^2 / mu^2$ - value of impact parameter, at which the particle deviates on perpendicular. We shall note, that rmin has not the relation to a low limit of integration, which used Landau. At the solution of this problem all collisions with r < ρ_{max} are taken into account exactly. The force of friction was used for a comparison with the computer experiment rezult. The value of average friction force in computer experiment was determined by change of a total moment of electrons per unit of time. The comparison received in computer experiment of friction force with the analytical form for the friction force in the binary collisions approximation permits to determine value rmax. The analytical form for the friction force does not divergen for small impact parameters. Therefore, this problem is convenient for the Coulomb factor determination (Coulomb logarithm), describing the phenomenon of multiple collisions in a Coulomb plasma.

3. Results of many particles dynamics modeling

Lot of code run with various parameters of computing and physical model was carried out. It was vary number of electrons (z+1)*n* , umber of multiply charged heavy ions *n* , relation of weights m_i/m_e , evolution time t_0/τ_{ei}, where $\tau_{ei}=N_i^{-1/3}/(3T_e/m_e)^{1/2}$ is average electron time of flight over the mean distance between ions. Electron temperature T_e has sense a conditional characteristic of a kinetic energy size in this problem. Parameter characterizing the degree of plasma coupling (imperfection, non-ideality) $\delta=2e^6N_e/T_e^3$ (at z=1) for these parameters is equal $\delta=6(10^{-4},\ 10^{-7})$, the number of particles in the Debye sphere varies from 2 up to 60. As a result of account on speed of loss of a electron impulse was calculated mean friction force, further were calculated the Coulomb logarithm and upper limit of integration on impactparameter.

N run	z	Λ	Λ^*	r_D/r_{ii}	ρ_{max}/r_{ii}	t_0/τ_{ei}	m_i/m_e	(z+1)n
1	1	3,9	3,2	1,1	0,5	20	10^2	256
2	1	3,9	3,2	1,1	0,7	20	10^2	1024
3	1	3,9	3,2	1,1	0,7	20	10^2	4000
4	1	4,9	3,7	1,5	0,45	50	10^3	1024
5	1	6	4,5	2,2	0,5	20	10^6	1024
6	1	6,3	5,0	2,5	0,6	100	10^3	1024
7	1	7,4	5,9	3,5	0,8	100	10^3	1024
8	2	5,6	4,0	1,9	0,4	50	10^3	766
9	8	6,7	4,3	2,7	0,4	80	10^4	674
10	8	6,7	5,1	2,7	0,6	100	10^5	674
11	8	6,7	5,1	2,7	0,6	25	10^5	3600
12	80	6,7	5,2	2,7	0,6	25	10^5	3969

The case of unitary charged ions is studed more careful (runs 1-7). The runs 1-3 are carred out for systems with different namber of particles and runs 4-7 for different degree of plasma coupling. For the all cases the calculated cut off impact parameter is equal approximately to half of interpartial distance. The being present differences have not systematic character. It depends from a modeling parameters - numbers of particles, weight of ions, radius of Coulomb potential modification at close distances, time of the process.

To determination of stationary values of the collision characteristics (Coulomb logarithm, cut off parameter) they were calculated for each moment of a time. The duration of evolution was chosen such, that the change of a electron impulse was not less 10 % from initial value.

4. Conclusion

The Boltzmann collisions integral do not apply for a plasma because of multiply character of a Coulomb collision. But by choice of a cut off upper limit of Coulomb cross sections integration, equal to interpartial distance, good agreemente with results of modeling from ab initio is received. In the known formulas for a kinetic factors, using frequency of collisions, it should use a significance of a Coulomb logarithm, which is received by choice of the upper limit integration equal to interionic distance.

References

[1] Mayorov S.À., Òkachev À.N. and Yakovlenko S.I. Physica Scripta, **51**, 498(1995).
[2] Landau L. Phys. Z. der Sow. Union, **10**,154(1936)
[3] Koga T. Introduction to kinetic theory stocastic processes in gaseous systems. Oxford Pergamon Press, 1970.
[4] Vlasov A.A. Journ. of Phys. **9**, 26 (1945); Journ. of Phys. **9**, 190 (1945).
[5] Persico E. Monthly Notices, Roy. Astron. Soc., **96**, 294 (1926).
[6] Langmuire L. Proc. Nat. Acad. **14**, 627(1928).
[7] Davidov B.I. Phys. Z. der Sow. Union, **9**, 433(1936).
[8] Druveystein M. Physica, 562(1938).
[9] Spitser L. Monthly Notices, Roy. Astron. Soc., **100**, 396 (1940).
[10] Cohen R.S., Spitzer L., Routly P. Phys. Rev. Vol. **80**, No. 2, 230(1950).
[11] Cowling T.G. Proc. Roy. Soc. **A, 183**, 453(1945).
[12] Chandrasekhar S. Astrophys. J. **94**, 511(1941).
[13] Hockney R. and Eastwood J. Computer simulation Using Particles. (McGraw-Hill, NY 1981)
[14] S.A.Maiorov//Bulletin of the Lebedev Physics Institute. No.9,(1997)

Address: Institute of General Physics of Russian Academy of Science, Vavilov Street 38, 117942 Moscow, Russia.
E-mail: mai@kapella.gpi.ru + may@mikron.ru; Tel. +7 (095) 1350247; +7 (095) 5369476