# Lecture 11 Physics 101

Please turn in Homework!

First, an announcement: the first exam (yikes!) will be two weeks from today, on October 11. The exam will be in class, closed book/notes, but you will be provided with a formula sheet. The exam will cover all material in the textbook up through and including Chap. 8. More details, practice exams, etc. will be provided closer to the exam.

To begin today's lecture, I want to visit the lecture ticket and test out the question with real equipment! Here, I have a string attached to a bar, which is itself attached to a weight, off of which another string is attached. We are going to do two things: first, I will quickly yank the bottom string, and then I will slowly pull the bottom string. So, I have two questions for you:

1) When I yank the bottom string quickly, which string will break first?

a) top  b) bottom  c) both at same time

2) When I slowly pull the bottom string, which string will break first?

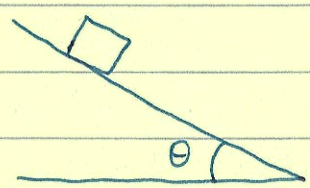
a) top  b) bottom  c) both at the same time

Talk to your neighbors for a couple minutes and see if you can come to a consensus!

Let's think about these two cases for a second before we try it out. For the set-up in which we yank the bottom string, we are imparting a large force very quickly. Even though it is a large force, because of the large mass of the weight, its acceleration is small, which means that the acceleration of the string above it is small. Only the string at the bottom has a large acceleration, and so it will break first.

By contrast, if we slowly pull the bottom string, then the bottom string, weight and top string accelerate together. The top string additionally is pulled by the weight of the, uh, weight, so the tension on the top string is greater than that of the bottom string. With enough pull at the bottom, the top string will break first. Let's test this out!

We've talked about friction on Wednesday and introduced the coefficient of static friction $\mu_s$. One way to determine $\mu_s$ is to just see what the minimum force you must apply to get a block moving, and compare that force to the normal force/weight of the block. This technique requires measuring forces accurately, which we may not be able to do easily. Instead, I want to introduce another method here that is much simpler and only requires measuring one angle. Here's the setup: I am going to put a block on an incline/ramp whose angle with respect to the horizontal I can vary, like so:




Now, I tilt the ramp, increasing $\theta$ juuuuust until the block starts to move. That angle, $\theta_{min}$, then tells me information about the coefficient of static friction, $\mu_s$.

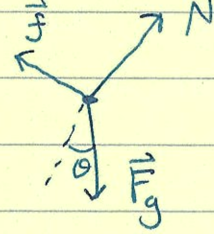
We'll analyze this in a second, but I want to first ask you to think about how $\theta_{min}$ is related to $\mu_s$. Is $\mu_s$ equal to:

a) $\mu_s = \cos\theta_{min}$ b) $\mu_s = \sin\theta_{min}$ c) $\mu_s = \tan\theta_{min}$

d) some other relation

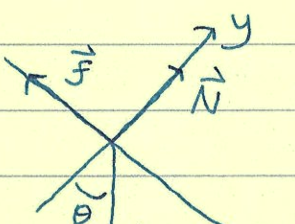
Don't fully analyze the problem; just think about limiting cases! I'll give you a second to talk to your neighbors.

Okay, let's analyze this system now. As always in this game, we draw a free-body diagram for the block:




Note the Normal force is <u>always</u> perpendicular to the ramp, and the block would want to slide down the ramp, so friction force points <u>up</u> the ramp.

As the free-body diagram demonstrates, this is manifestly (= obviously) a two-dimensional system, so we need to identify our axes appropriately, in a way to simplify the physical description. The physics cannot depend on the coordinates we use, but we can exploit properties of the system. First, our goal is to understand the force of friction, $\vec{f}$, so it might be easiest to align the friction force with an axis. Correspondingly, the Normal force, $\vec{N}$, would also point along an orthogonal axis. So, we will set up coordinates as:




, where x-axis points down the ramp and y-axis is perpendicular off the ramp.

With this coordinate system, we can now write the vectors in component form. We have:

$\vec{N} = N\hat{j}$ , $\vec{f} = -f\hat{i} = -\mu_s N\hat{i}$ , $\vec{F}_g = mg\sin\theta\,\hat{i} - mg\cos\theta\,\hat{j}$

Note also that we are assuming that the block is at rest, and so its acceleration is $\vec{a}=0$, and net force is also 0. Therefore, the net force in each dimension must be 0. As a vector equation, we have:

$$\vec{N} + \vec{f} + \vec{F}_g = \vec{F}_{net} = 0 = (mg\sin\theta - \mu_s N)\hat{i} + (N - mg\cos\theta)\hat{j}$$

or that: $N - mg\cos\theta = 0$,
$mg\sin\theta - \mu_s N = 0$.

This requires that $N = mg\cos\theta$, and plugging this into the second equation, we have

$$mg\sin\theta = \mu_s mg\cos\theta \quad \text{or that} \quad \mu_s = \tan\theta_{min},$$

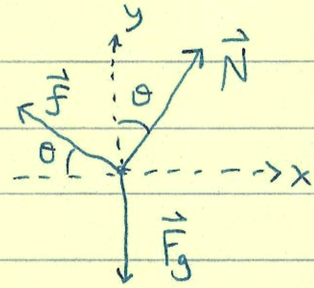
where $\theta_{min}$ is the minimum angle at which the block slides.

Whenever, you get a result, you should always ask if it makes sense. Does anyone here ski, mountain climb, or hike? The tools you use for each of these activities are optimized to increase or decrease friction. First, if you ski, you wax your skis to decrease the coefficient of friction so you can ski faster, as well as ski (that is, actually move) on slopes with small inclines. If $\theta_{min}$ is small, $\tan\theta_{min}$ is also small and $\mu_s$ is small. Makes sense.

By contrast, if you are rock climbing, you don't want shoes covered in Teflon®, you want sticky, rubber shoes. You want to stick to steep walls, making $\theta_{min}$ as large as possible. If $\theta_{min}$ is large, then $\mu_s$ is large. Again, makes sense.

For the final part of this lecture, I want to revisit the system we just studied, to explicitly demonstrate that indeed the physics is independent of our description. Instead of aligning our axes with the slope, we'll just keep the axes horizontal and vertical. This will lead to different intermediate steps, but the final result will be identical. Further, it's important to know how to solve physics problems in many different ways. Different solutions to a problem provide different insights into how physics is working and manifesting itself, and can lead to a deeper understanding, even for the most mundane of problems.

So, with that motivation, let's redraw our free-body diagram with our new coordinate basis.




In this coordinate basis, the components of the vectors are:

$$\vec{F}_g = -mg\,\hat{j}\,,\quad \vec{N} = N\sin\theta\,\hat{i} + N\cos\theta\,\hat{j}\,,\quad \vec{f} = \mu_s N\cos\theta\,\hat{i} + \mu_s N\sin\theta\,\hat{j}$$

As before, there is no acceleration in either dimension so Newton's second law implies:

$$\hat{i}:\ N\sin\theta - \mu_s N\cos\theta = 0\,,\quad \hat{j}:\ -mg + N\cos\theta + \mu_s N\sin\theta = 0$$

Now, with this coordinate system, the $\hat{i}$ equations immediately produce:

$$\mu_s = \tan\theta_{min}$$

Recall that earlier we had found that $N = mg\cos\theta$, when our coordinates were oriented along the ramp. Does that still work in this case? Solving for $N$ in the $\hat{j}$ equation, we have

$$N = \frac{mg}{\cos\theta + \mu_s\sin\theta}.$$ With $\mu_s = \tan\theta$, note that

$$\cos\theta + \tan\theta\sin\theta = \frac{\cos^2\theta + \sin^2\theta}{\cos\theta} = \frac{1}{\cos\theta}.$$ Then

$N = mg\cos\theta$, as earlier! So, indeed the physics is independent of our description of it. That's it for today; have a good weekend!