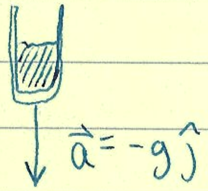
# Lecture 12 Physics 101

Hope you had a good weekend! Please turn in homework. Also, remember that we will have our first midterm exam on October 11! More details to come, but it will be closed book, closed notes, but you will be provided with a formula page with relevant equations. Stay tuned...

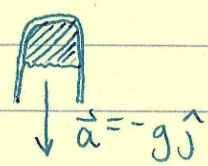
I want to impress my son, so I've thought of a fun experiment. We all know that water will sit in the bottom of an upright glass. Indeed, if this were not the case, life would be much more difficult! Anyway, that's not the experiment. What I want to do is keep the water in the glass when it is upside down. Is this possible? Clearly, I can't just turn the water glass upside down and keep water in it, because that is how you drink. So let's go systematically through systems that would keep the water in the glass.

First, let's pour water in the glass and just drop it, upright, on the ground. Does the water stay in the glass in this case? As always, we ignore air resistance. As the glass an water fall, they <u>both</u> accelerate at $g$:



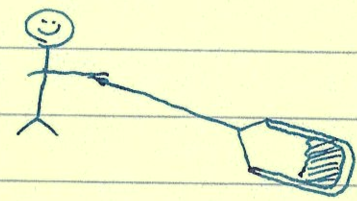
Because the glass and water start together and accelerate at the same rate, they stay together as they fall. Thus, the water stays in the glass as it falls.

This correspondingly tells us how we can keep ~~gla~~ water in the glass when it is upside-down. Pour water into a glass when you are on an airplane, jump out, and then turn the glass over! The water and glass (and you) will all accelerate at g, and so the water will stay in the glass:



So we've succeeded! We just need to accelerate the glass/water downward at g, i.e., in free fall, and the water will stay in the glass. However, demanding that we do this by jumping out of a plane isn't very practical.

Let's think of another way to get the glass upside-down without the water spilling. We could also swing the glass with water in it in a big, overhand circle. At the bottom of the circle, the glass is upright with the water in it, while at the top of the circle, the glass will be upside down with the water in it, just like we want. So the physical set up is:



where I have (poorly) drawn a picture of me attempting to twirl the glass in a circle.

So, is it possible to twirl a glass in an overhand circle so that the water does not fall out?

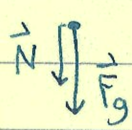
a) not possible b) yes, possible at any twirl rate

c) yes, possible but there is a minimum twirl rate

Talk to your neighbors for a second!

For the water to stay in the glass, then necessarily the water must stay in the glass when it is at the top of the circle:

What are the forces on the water? Of course gravity, but also (possibly) a normal force from the glass:




There are no other relevant forces, so this suggests that the water is accelerating, by Newton's second law!

We had already argued that for the water to stay in the glass, the acceleration $a_y$ of the water must be (at least) $g$. Using Newton's second law we find:

$$ma_y = N + mg \quad \text{or that} \quad a_y = \frac{N}{m} + g \geq g,$$ as normal force and gravity act in the same direction.

Additionally, the glass/water system is traveling in a circle, so we know how to interpret this accleration: it is centripetal acceleration, where its magnitude is

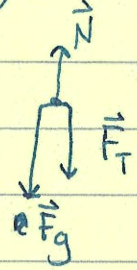
$$a_{cent} = a_y = \frac{v^2}{r},$$ where $v$ is the tangential velocity of the glass/water, and $r$ is the length of the string I am swinging (the radius of the circle). So, to keep the water in the glass, we must have that:

$$a_y = \frac{v^2}{r} \geq g \quad \text{or that} \quad v \geq \sqrt{rg}.$$

That is, there is a minimal speed below which the water will fall out of the glass and above which it will stay in.

Let's test this out! I have here a cylinder on a tray and I will fill the cylinder with water and we'll observe what happens when I swing it above my head.

Let's continue to analyze this system. In particular, what is the tension in the string at the top of the circle? To study this, let's consider the free-body diagram for the glass at the top:




There are only three forces acting on it at the top: its weight (of course), the tension in the string that twirls it, and the normal force of the water. The normal force of the glass on the water is a force pair with the normal force of the water on the glass, and so this force on the glass acts in the vertical direction, upward. By Newton's second law, the sum of these forces is responsible for the centripetal acceleration of the glass:

$$T + mg - N = ma_{cent} \geq mg$$

In the inequality on the right, we have simply noted that the centripetal acceleration at the top of the loop must be at least the acceleration due to gravity, $g$. So, rearranging the inequality, we find that the magnitude of the tension force is at least the magnitude of the Normal force:

$$T \geq N$$

We argued earlier that the minimum normal force on the water/glass by the glass/water is $N=0$. That is, the tension $T$ is, well, anything and the glass can still travel in a circle.

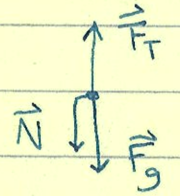
However, our analysis assumed that there was a tension force at all! What happens if it is no longer true that

$$a_{cent} = \frac{v^2}{r} \geq g?$$

That is, what if the velocity of the glass is too small at the top of the loop? In that case, there is no normal force, the string goes slack, and the only force on the glass is gravity. That is, the glass would enter free-fall, and just travel in a parabola, rather than a circle.

Let's test this out! Now this is much more dangerous than the earlier demonstration because you very really lose control once an object is in free-fall. However, I am here to sacrifice my body to science, so let's do it!

It's also interesting to consider the forces that are acting on the water/glass at the bottom of the circle. At that point, the forces on the glass are:




Now, if the glass is traveling in a circle, the sum of these forces are responsible for centripetal acceleration:

$ma_{cent} = m\frac{v^2}{r} = T - N - mg$ , or that the tension in

the string is: $T = m\frac{v^2}{r} + N + mg$.

This is fascinating: we can spin/twirl the glass faster and faster (increase v) and eventually the tension will be so large that the string will break!

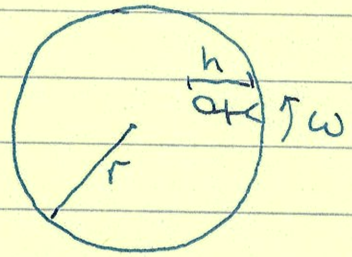
Let's try this out! Actually, no way, this is <u>much</u> too dangerous! :)

Before we continue, I want to have you think about something. If the centripetal acceleration at the top of a loop of radius r is just g, then what could the expression for the velocity at the bottom of the loop be? What changes from when the glass is at the top to when it is at the bottom?

In a related vein, consider swinging on a, well, swing. Is it possible for you, with no one pushing you to ever "swing over the bar"? Can you <u>ever</u> pump your legs hard enough to provide high enough speed such that your velocity at the tippy-top provides enough centripetal acceleration to equal g? How fast would you have to be going at the bottom?

A few more notes before we end for today. One aspect of roller coasters is the feeling of weightlessness at the top of a loop. "Weightlessness" simply means that the only force acting on you is gravity, so you are accelerating at g. One feature of the particular feeling of weightlessness is "butterflies in your stomach". This is due to your organs literally floating in your body when you are weightless. When you are standing on ground, your organs are held in place by a matrix of ligaments and such, but when weightless, the tension on the matrix vanishes, leaving your guts a-floating.

Additionally, this property of circular motion can be exploited to simulate the force of gravity. There's a famous scene in "2001: A Space Odyssey" in which David Bowman is running on the inside of a revolving cylinder as such:




Say the revolving cylinder has radius r and is rotating with angular speed ω. We then know the centripetal acceleration of the cylinder:

$a_{cent} = \omega^2 r$. If this equals g, then, the acceleration of the cylinder would be the same as the acceleration due to gravity on Earth. However would this apparatus actually simulate the gravity we know and love? Think about it!

See you wednesday!