# Lecture 13 Physics 101

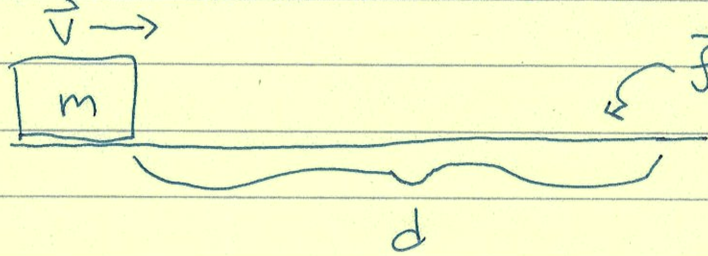
Welcome back from a Tuesday away from lecture! Please turn in homework. Reminder that the exam will be in class on October 11. It will be closed book and closed notes, but you will be provided with an equation sheet. I'll post the equation sheet to Moodle soon (like after class).

Beginning this lecture, we will start our foray into conservation laws, their consequences, and utility for understanding physical systems. I had introduced conservation laws in the first lecture from asking why we can trust our memories. The physics answer to this is that the laws of physics; i.e., the rules that govern how we engage with Nature and our environment in particular, do not change in time. That is, things we learned yesterday (I hurt my hand by touching a hot stove) can be applied to actions tomorrow (don't touch a hot stove). I had also mentioned then that this implied the existence of a symmetry: a transformation that we can perform on a system that leaves it unchanged. More precisely, a ~~sy~~ symmetry transforms a system to itself. In the case of trusting our memories, the symmetry action is time translation: the laws of physics are unchanged by travel (translation) through time. Additionally, I had argued that there should be a single quantity that is a measure of this time translation symmetry. That is, if time translation is a perfect symmetry, then this quantity is conserved, its value does not change in time. This intricate relationship between symmetries and conservation laws is called Noether's Theorem and the conserved quantity associated with time translation invariance is energy.

We denote energy by E and we will define it (for now) as the measure of an object's or system's ability to perform a task. This definition is consistent with our colloquial use of "energy". If you "have no energy", then you can't even perform simple tasks. Today, we will just study the energy of single object or particle systems, which will simplify our task for determining what this "energy" is.

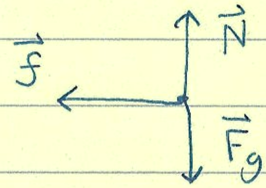
Historically, like during the Renaissance/Enlightenment period, a way to measure the energy of an object was the following. A lead ball of mass m was thrown with velocity $\vec{v}$ at a chunk of clay. The ball correspondingly smushed the clay, embedding itself a distance d into the clay. The distance d was thus a measure of the lead ball's ability to perform a task; that task being deforming the clay. We'll study a similar system, more familiar for what we have studied, to identify the energy of a moving object.

What we will do is the following. We will give a block of mass m a velocity $\vec{v}$, and then it will travel over a surface which has friction. This friction force will accelerate the block, eventually stopping it. We would like to determine the block's ability to slide over the surface. We will measure this ability, i.e., the block's energy, by the distance d it travels over the surface. So, the set-up is:




Before we study this system, I want to make a couple of notes. First, Energy is a <u>scalar</u> quantity: it has only a magnitude, and has no direction. From our set up, this makes sense: the distance d is just a distance (magnitude) and not a vector (displacement). <u>Whatever</u> direction $\vec{v}$ is, we want to stop it, so direction is irrelevant.

To study this system, let's recall the kinematic equations and Newton's $2^{nd}$ law. The free-body diagram for the block when it is on the surface with friction is:




The block is not accelerating vertically, so $\vec{N} = -\vec{F}_g$. Newton's Second Law in the horizontal direction is:

$\vec{f} = m\vec{a} \Rightarrow -f\hat{\imath} = m a_x \hat{\imath}$, or that acceleration $a_x$ is

$a_x = -\frac{f}{m}$. We could relate $f$ to normal force and therefore weight, but we won't here.

With the initial velocity vector $\vec{v} = v\hat{\imath}$, we have the kinematic distance equation:

$d = \frac{1}{2} a_x t^2 + vt$, as the block slides a distance $d$ along the surface. This is written with time explicitly, but we can eliminate that using the kinematic equation for speed:

$0 = a_x t + v$, where we note that the final velocity is $0$.

I want to emphasize that these kinematic equations can be used because acceleration $a_x$ is constant:

the friction force $f$ is constant. Then, the time over which the block slides is:

$$t = -\frac{v}{a_x}.$$

Plugging this into the equation for distance, we have:

$$d = \frac{1}{2} a_x \left(-\frac{v}{a_x}\right)^2 - v \frac{v}{a_x} = -\frac{1}{2} \frac{v^2}{a_x}$$

We had found that $a_x = -\frac{f}{m}$ earlier, so this is also

$$-\frac{1}{2} \frac{v^2}{\left(-\frac{f}{m}\right)} = d \Rightarrow \boxed{\frac{1}{2} m v^2 = f d}$$

This relationship is immensely profound. The expression on the left, $\frac{1}{2} m v^2$, is exclusively written in terms of the block's properties. It is called <u>kinetic energy</u> because it is a measure of the energy due to the block's motion:

$$K = \frac{1}{2} m v^2$$

The expression on the right, $fd$, is exclusively written in terms of how the surface acts on the block. The surface exerts a force $f$ on the block over a distance $d$. This force is responsible for reducing the kinetic energy of the block from $\frac{1}{2} m v^2$ to 0. As such, we say that the surface did <u>work</u> on the block equal to:

$$W = f d.$$

We'll explore this later and more precisely, but work done by a force changes an object's kinetic energy. Specifically,

$$W = \Delta K, \text{ called the Work-Energy theorem.}$$

Now, we said some words that if energy is conserved, then the laws of physics should be independent of time. The work-energy theorem is a statement of conservation of energy: kinetic energy can be transformed into some other form of energy by exerting work, but can't disappear into the aether. Newton's second law is a law of physics, so if energy is conserved, it should somehow follow from $W = \Delta K$. Let's see how this is done in our example.

We had derived: $\frac{1}{2}mv^2 - fd = 0$, which is just the statement of the work-energy theorem, with everything to one side. If this is true at <u>any</u> time, then it has no time dependence or its time derivative is also zero. That is, the statement of time independence is:

$$\frac{d}{dt}\left(\frac{1}{2}mv^2 - fd\right) = 0.$$

Let's take derivatives! The mass $m$ of the block is constant, so the only relevant derivative of kinetic energy is of the $v^2$ term. We find, using the chain rule,

$\frac{d}{dt} v^2 = \frac{dv^2}{dv} \frac{dv}{dt} = 2va$, where we note that $\frac{d}{dv} v^2 = 2v$
and $\frac{dv}{dt} = a$, acceleration.

Then, the time derivative of the kinetic energy is,

$$\frac{d}{dt} \frac{1}{2} mv^2 = mva.$$

Next, the time derivative of $fd$ is equally simple. By assumption, the friction force is a constant, so only $d$ might depend on time. However,

$\frac{d}{dt} d = v$, just the velocity. It then follows that

$\frac{d}{dt}(fd) = -fv$. (Minus sign because as $d$ increases, speed decreases because friction slows the block)

Using these results, we have

$$\frac{d}{dt}\left(\frac{1}{2}mv^2 - fd\right) = 0 = mva + fv$$

The velocity $v$ appears in both terms, so we can safely cancel it out, producing:

$$ma = -f$$

which is just Newton's second law for a friction force. This demonstrates that, in a well-defined way, Newton's 2nd law is very literally derivative of conservation of energy. As such, we consider conservation of energy more fundamental than Newton's $2^{nd}$ law.

We will provide a more precise definition of work, the work-energy theorem, and demonstrate how Newton's second law follows as a vector equation from conservation of energy in the coming lectures.

For the remainder of this lecture, I want to use this new idea of kinetic energy to understand a feature of my research. The Large Hadron Collider (LHC) in Geneva, Switzerland, accelerates and collides protons at enormous (relative) energies. We'll attempt to get a sense for how large the energy of an individual proton is at the LHC. First, the unit of energy in SI is called the Joule, after James Joule, a Scottish engineer. By the work-energy theorem Note that the units of the Joule are:

$$[J] = N \cdot m = kg\, m^2 s^{-2}$$

In Joules, the kinetic energy of a proton at the LHC is about $10^{-6}$ J. (More useful unit of energy in particle physics is the electron-Volt, for which the protons carry about $10^{13}$ eV of energy.) Objectively is this a lot of energy?

$10^{-6}$ J is a small number, but let's relate it to more everyday energies. For example, let's consider the kinetic energy of a flying mosquito. The mass of a mosquito is about

5 mg or $5 \cdot 10^{-6}$ Kg. At top speed, a mosquito can fly at about $1/2$ m/s, so their kinetic energy is:

$$K = \frac{1}{2}(5 \cdot 10^{-6}) \cdot (.0.5)^2 \, J \cong 6 \times 10^{-7} J$$, very close

to the kinetic energy of a single proton at the LHC!
I want to emphasize the scale here. A mosquito contains about $10^{20}$ protons, and yet the LHC packs the same energy of a mosquito into 1 proton!

The energy of a mosquito may still be a bit abstract, so let's try another comparison. Your hand (rather, one of them) is about 0.5% of your body weight. I weigh about 80 kg (times g), so the mass of my hand is about

$$m_H = 0.5 \cdot 10^{-2} \cdot 80 = 0.4 \text{ Kg}.$$

If the kinetic energy of your hand is K, then its velocity is:

$$K = \frac{1}{2} m_H V^2 \Rightarrow V = \sqrt{\frac{2K}{m_H}}$$

Plugging in $K = 10^{-6}$ J and $m_H = 0.4$ Kg, the velocity of one of your hands nesessary to have kinetic energy equal to one proton at the LHC is:

$$V = \sqrt{\frac{2 \cdot 10^{-6}}{0.4}} \, m/s \cong 2 \cdot 10^{-3} \, m/s$$, or a couple millimeters per second.

This is the rate of a slow clap, again, contained in a single proton at the LHC.

The LHC doesn't collide individual protons together; rather bunches of about $10^{11}$ protons are collided. There are about $10^{10}$ people on Earth, so there is more energy in the protons at the LHC than if every human on Earth simultaneously clapped.

That's it for today! Have a good Wednesday!