Lecture 17 Physics 101

Welcome back! I hope you had a relaxing weekend. Homework assigned last Wednesday is due today. There is homework assigned today (due Wednesday) and Wednesday (due Friday) for this week. No homework will be assigned over fall break. Woohoo!

As I mentioned last week, the lecture before fall break has traditionally been devoted to discussing the Nobel Prize in Physics. Both because I like to buck tradition and I have philosophical issues with the Nobel, I am asking you to decide what I should lecture on Friday. I have made a poll on this week's entry on Moodle where you can vote for a lecture topic. The choices are:

- The 2019 Nobel Prize

- The Nobel Prize in History, where I will talk about a history of the Nobel in Physics and my plethora of problems with it

- My Research, I'll spend 50 minutes boring you with what I find extremely exciting :)

- Einstein's Theory of Gravity, General Relativity to provide perspective and consequence for Newton's theory of Gravity we will discuss Wednesday, and

- Something else

Please vote soon; polls will close Wednesday evening because I need time to write the lecture.

Enough logistics; let's get back to physics. We have discussed energy, its conservation, kinetic vs. potential, and work, and today we're going to tie together some loose ends before moving on. Last Wednesday, we had discussed the notion of a <u>conservative force</u>: a force for which the work that it does exclusively comes from expending potential energy. The conservative forces we will focus on in this class are gravity and Hooke's law (springs), and we used this idea to determine the height to release the ball to go around the loop-the-loop last week. What if there are non-conservative forces in the game; i.e., friction? Conservation of energy still holds, we just have to account for the work done by the non-conservative forces.

Conservation of energy is simply the statement that the energy measured at an initial time E_i

is equal to the energy measured at some later time E_f:

$$E_i = E_f$$

With only conservative forces in the ballgame, this can be restated through a sum of kinetic and potential energies:

$$K_i + U_i = K_f + U_f \quad \text{(conservative forces only)}$$

When analyzing a system in which the only forces are gravity and springs, this form of energy conservation is most useful.

With friction or other non-conservative forces around, some of that initial energy can be lost to heat, sound, etc., and not manifest as kinetic or potential energy in the final state. Therefore, accounting for this energy moved from a "useful" form (kinetic, potential) to a "useless" form (heat, sound), we account for the work that non-conservative forces did in going from the initial system to the final system:

$$K_i + U_i + W_{non\text{-}cons} = K_f + U_f$$

Note that energy is still conserved, just not strictly conserved as kinetic and potential energy exclusively. Also most (all?) work by non-conservative forces is negative; e.g., friction slows an object. So, this implies that, in general, when non-conservative forces are around, initial kinetic and potential energies are larger than their final values.

Another key concept with energy is power, or energy used/delivered per unit time. At its simplest, power P is just the time derivative of the energy of some object:

$$P = \frac{dE}{dt}.$$

A single object like a car, horse, plane, etc., is not a closed system, so its energy does not need to be conserved; that is it can change in time. In our everyday experience, it is power that makes a task challenging. Expending a lot of energy very quickly is more difficult than expending the same energy more slowly.

Let's derive another relationship of power using the work-energy theorem. The work-energy theorem states that

$$\Delta E = \int_a^b F\,dx,$$

that is, the energy of an object (spring, car, apple, etc) changes if a force is applied over some distance: $x \in [a, b]$.

Differentially, this relationship is:

$$dE = F\,dx$$

Now, to relate this to power, we just divide by the infinitesimal time dt on bothsides. (Nota Bene: I am not a mathematician, so questionable manipulations with infinitesimals are all cool! ☺) That is:

$$\frac{dE}{dt} = P = F\frac{dx}{dt} = Fv$$

Or, power is force times velocity! Now, I've been working in one-dimension, so to generalize to multiple dimensions, we need a dot product between force and velocity..

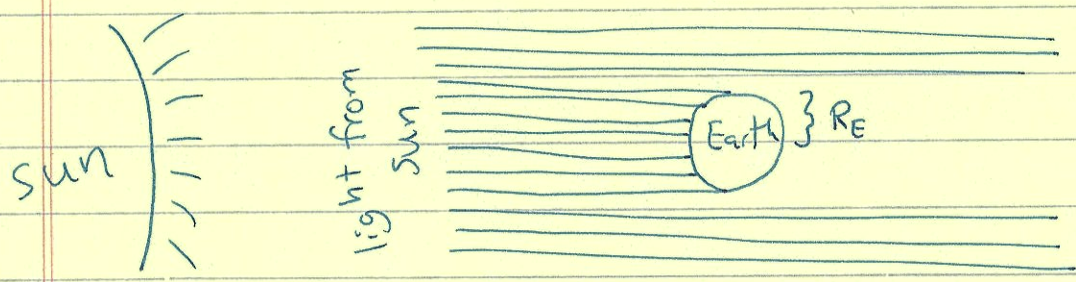
$$P = \vec{F}\cdot\vec{v} = Fv\cos\theta.$$

The units of power in SI are Watts, after James Watt who invented the steam engine. A Watt is, not surprisingly, one Joule of energy per second, both SI units themselves. Another unit of power you might have heard of is horsepower, which interestingly was introduced by James Watt to compare the output of his steam engines to draft horses. The "horsepower" used in the United States for car energy output, for example, is 745.7 watts.

Now, with this definition of Power, I want to estimate the total power that is accessible from the sun on Earth. Solar power is increasingly becoming an important ~~an~~ renewable resource, providing energy to us to charge our phones, run our trains, or heat our lecture halls. For the rest of this lecture, I want to estimate the possible solar power that we can harness on Earth.

First, I'll tell you a couple of things. The power consumption of the entire Earth is about 10^{13} Watts, 10 TeraWatts. That means that every second, 10^{13} Joules are needed for everyone on Earth to heat their homes, run their Teslas, or cook their dinner. So, we're going to attempt to answer the question of if solar power can account for these 10^{13} Watts.

The amount of power from light emitted by the sun incident on Earth is about 1000 Watts per square meter. That is, when the sun is directly overhead on a clear day, 1000 Joules of light energy hits ~~the~~ a square meter of ground every second. For some context, a standard lightbulb in your house might use about 60 Watts of power to run. (LED bulbs use <u>much</u> less power for the same light output, though.) So, ~~to~~ as a first step in getting to our answer, let's consider how much ~~light~~ solar power is incident on Earth at any time. The trick to answer this is to introduce the notion of cross-sectional area. The sun shines light on Earth, and the cross-sectional area is the size of the shadow that Earth produces:



The size of Earth's shadow is equal to the area of a circle with Earth's radius; this is called the cross-sectional area because if you cut Earth in half (i.e., made a cross-section), the area of the surface you opened up would be:

$$\text{Area} = \pi R_E^2$$

With the radius of the Earth $R_E \cong 6000\,\text{Km} = 6 \cdot 10^6\,\text{m}$, the cross-sectional area of Earth is:

$$\text{Area} = \pi \cdot (6 \cdot 10^6)^2\,\text{m}^2 \cong 10^{14}\,\text{m}^2$$

Again, as everything in this game, orders of magnitude are sufficient. Using this, to find the total solar power incident on Earth, we multiply this area by the 1000 W/m² to find:

$$P = 1000\ \mathrm{W/m^2} \cdot 10^{14}\ \mathrm{m^2} \cong 10^{17}\ \mathrm{W}$$

This would seem to be totally enough to power all of Earth, with orders of magnitudes to spare. But, there's a catch. To capture *all* of this power, we would have to cover the *entire* Earth in solar panels, the sky would *always* have to be clear, and solar panels would have to be 100% efficient. But none of these are true, so we need to incorporate realistic numbers in an estimate.

First, the efficiency of commercial solar panels is about 10%. That is, for an incident power P on a solar panel, only about $1/10$ of that power can actually be turned into electricity to power a toaster. So, out of the 10^{17} W of solar power incident on Earth, we can only extract about 10^{16} W for our use.

Now, we can't really hope to cover the ocean with solar panels. Oceans cover about 70% of Earth's surface, or land is only about 30% of Earth's surface. Further, clouds cover about 70% of Earth's surface at any given time, so of the 30% of land, only about 30% of it has a clear shot of the sun. 30% of 30% is about 10% again, so restricting solar panels to be on land means that there is only about 10^{15} W of power for our use.

Continuing, we can't actually cover all land with solar panels. If we did, no light would hit the ground, so there would be no farms, no forests, no fields. However, we could imagine that, say, a solar panel was installed on the roof of every building on Earth. This eliminates or at least minimizes the further footprint on the environment. As an estimate of the area of roofs on Earth, the total fraction of land area that is urban is about 3%. Of course, all of an urban area isn't just roofs, so perhaps a tenth of ~~an~~ urban areas could be covered in solar panels. Including this factor of about 0.3% or 3/1000, the total area where solar panels could be reduces the total solar power accessible to us~~e~~ would be about

$$P_{solar} \approxeq 3\times10^{12}\,W.$$

There may be further constraints on the total solar power that can be harnassed, like infrastructure issues, but even now, we have fallen below ~~at~~ the level of covering all power need of Earth. Solar can't be all if we are to divest energy consumption from petroleum to renewable sources.

That's it for today; on to gravity on Wednesday!