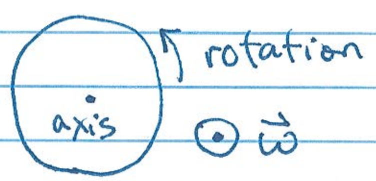
Lecture 28 Physics 101

Welcome back from the weekend! Please turn in homework!

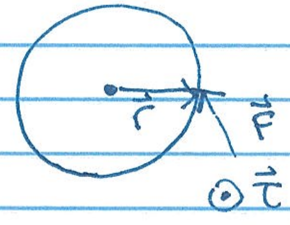
On Friday, we had introduced the angular velocity and torque as honest vectors, identifying the direction associated with them. This was nontrivial and unfamiliar from our analysis of vectors like velocity and force, because angular velocity and torque describe properties of rotation in a plane. Our solution to identifying a unique direction for these vectors was to note that they should encode information about the axis of rotation, and the direction of rotation about the axis of rotation. Given an axis of rotation, an object can rotate one of two directions, and so we identify the angular velocity vector as pointing in one of two directions down the axis of rotation. In practice we find the direction of $\vec{\omega}$ using the right-hand rule: point the fingers on your right hand in the direction of rotation, curl them in the direction of rotation about the axis, and your thumb points in the direction of $\vec{\omega}$:



We identified torque as the application of a force on an extended object that acts to change the rotational motion of that object. As such, there is a corresponding Newton's law for rotation:

$\vec{\tau} = I\vec{\alpha}$, where $\vec{\tau}$ is the torque, I is the object's moment of inertia, and $\vec{\alpha}$ is the angular acceleration.

Torque is not just a force: it depends on how and where on an object it is applied, with respect to the axis of rotation. For example, a bicycle tire where we apply a force as so:



Only the component perpendicular to the radial vector $\vec{r}$ of the force $\vec{F}$ acts to rotate the tire. This component can be extracted with the vector cross-product:

$$\vec{\tau} = \vec{r} \times \vec{F}, \text{ which has magnitude } \tau = rF\sin\theta,$$

where θ is the angle between $\vec{r}$ and $\vec{F}$. The direction of torque is also found from a right-hand rule. This time, the rule is: point the fingers on your right hand in the direction of $\vec{r}$, curl in the direction of $\vec{F}$, and your thumb points in the direction of torque/angular acceleration.

Now, with that long set-up and review let's discuss some consequences. To motivate today's lecture topic, I want to take a brief digression and discuss the the Sagsaywaman site, near Cuzco, Peru. Talk about Sagsaywaman and show pictures.

So how does one go about building a structure like Sagsaywaman? We don't know actually how it was done almost a thousand years ago, but we can answer the question of how would we build it now, knowing the laws of physics formalized as we have discussed. If we want to construct a wall to stand for a thousand years, then an obvious requirement is that we want the wall, and every element of the wall, to be at rest. Again, "at rest" means that the center-of-mass of an object is not moving. If the center-of-mass is not moving, then necessarily the net force on that object is 0:

$\vec{F}_{net} = 0$. For a wall, we need the net force of <u>every</u> element of the wall to be 0.

It's not enough to just have the net force on an object vanish for the wall to stand. A wall is a very boring object: it looks identical at <u>any</u> time after it is constructed. We know of systems for which their center-of-mass is at rest, and yet change in time. For example, we had studied two massive objects mutually orbiting their common center-of-mass through gravitation. There were no external forces, so the center-of-mass was at rest, yet the rotation <u>about</u> the center-of-mass meant that the system had some non-trivial time dependence. If you can back some time later, you would see the masses in a different position than originally. So, if a wall is not to move in any way, we must also require that it has no motion/rotation <u>about</u> the center-of-mass. If this is the case, then the net torque on the object is 0:

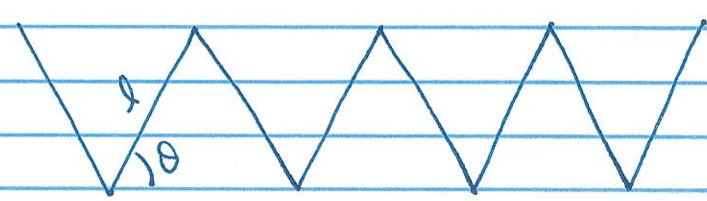
$\vec{\tau}_{net} = 0$. Again, for a wall, we need the net torque of <u>every</u> element of the wall to be 0.

So, compactly, we require that every element of the wall satisfies:

$\vec{F}_{net} = 0 \Rightarrow$ no motion of center-of-mass
$\vec{\tau}_{net} = 0 \Rightarrow$ no motion about center-of-mass

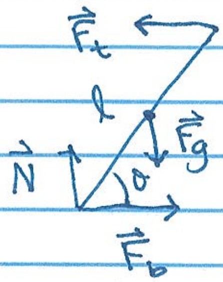
These two conditions are sufficient to enforce no motion of the object whatsoever. When they are both true, an object is said to be in static equilibrium. To a large extent, the job of a civil engineer, who designs roads, bridges, buildings, and other infrastructure, is to design a structure that efficiently and acceptably remains in static equilibrium.

As an example of a system in static equilibrium let's analyze a structure a bit simpler (:) than Saqsaywaman. We imagine the following: long, thin stones have been ~~been~~ placed in a circle and lean at an angle to come to a point with the next stone in the circle. That is, a segment of the circle looks like:



with the length of the stones l and the angle between the stone and the ground θ. The stones stay standing because the mutually exert forces on each other, similar to how a group of people can sit on each other's laps in a circle, with no one person holding up (or being held up by) more than one other person. For simplicity, we will assume no friction anywhere; can this ring stay standing?

Let's just focus on one stone and draw the forces exerted on it:



Here, I've denoted gravity acting at the center-of-mass of the stone, the normal force $\vec{N}$ from the ground, and additionally two normal forces, $\vec{F}_t$ and $\vec{F}_b$, exerted by the neighboring stones at the top and bottom of this stone. The ring of such stones can stay up if all forces acting on one stone are finite (or less than some structural maximum).

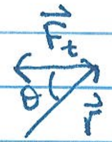
Let's first analyze $\vec{F}_{net}=0$. Note that two pairs of forces act exclusively vertically ($\vec{N}$ and $\vec{F}_g$) and two exclusively horizontally ($\vec{F}_t$ and $\vec{F}_b$). Therefore, $\vec{F}_{net}=0$:

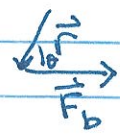
$$\vec{N}+\vec{F}_b+\vec{F}_g+\vec{F}_t=0$$

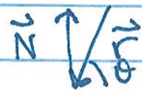
reduces to a statement about magnitudes:

$$N=F_g=mg, \quad F_t=F_b\equiv F.$$

That was easy; what about torques? We will analyze torques in two ways, about two different axes, but demonstrate that we get the exact same result, as we must. First, let's consider torques about the center-of-mass axis. About this axis, $\vec{F}_g$ has no torque, while the torques of the other forces are:

$\vec{F}_t$:  $\vec{\tau}\ \odot$, $\tau = \frac{l}{2} F_t \sin\theta = \frac{l}{2} F \sin\theta$

$\vec{F}_b$:  $\vec{\tau}\ \odot$, $\tau = \frac{l}{2} F_b \sin\theta = \frac{l}{2} F \sin\theta$

$\vec{N}$:  $\vec{\tau}\ \otimes$, $\tau = \frac{l}{2} N \cos\theta = \frac{l}{2} mg \cos\theta$

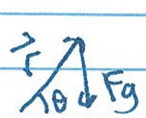
Accounting for the direction of torques, demanding the net torque be zero enforces:

$$\sum \vec{\tau} = 0 \Rightarrow \frac{l}{2} F \sin\theta + \frac{l}{2} F \sin\theta = \frac{l}{2} mg \cos\theta$$

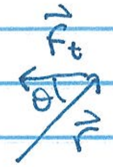
or that $F = \frac{mg}{2} \cot\theta$

That is, as $\theta \to 0$ (the stones get closer to the ground) the force they exert on each other increases. There will be some angle at which the force is so strong the stones will break, so they should be placed nearly vertically to minimize this force.

Finally, let's quickly analyze the net torque about another axis. Let's consider the axis at the bottom of the stone. Now, the normal force $\vec{N}$ and $\vec{F}_b$ have 0 torque as they are applied at the axis. The torques of the gravitational force and $\vec{F}_t$ are:

$\vec{F}_g$:  $\vec{\tau}\ \otimes$, $\tau = \frac{l}{2} mg \cos\theta$

$\vec{F}_t$:



$\vec{\tau}\ \odot$, $\tau = lF\sin\theta$

so the net torque about the axis at the bottom of the stone enforces:

$$\sum \vec{\tau} = 0 \Rightarrow Fl\sin\theta = \frac{l}{2} mg\cos\theta,$$ or that

$$F = \frac{mg}{2}\cot\theta,$$ which is the <u>same</u> requirement we found earlier!

That's it for today! See you wednesday!