Phys 101 Lecture 4

Welcome to lecture #4! Please turn in homework!

If we didn't finish the demonstration from last lecture we'll do that first. Otherwise, we're going to take this lecture to introduce another extremely powerful tool in a physicist's toolkit: order-of-magnitude estimation. Along with dimensional analysis, order-of-magnitude estimation can produce profound physical insights from very simple considerations. Many very famous physics papers are in essence nothing more than dimensional analysis and estimation.

This order-of-magnitude estimation, like dimensional analysis, is not designed to produce exact results but rather informs you of an answer within a factor of 10, or so. This might seem like it's not very useful as a factor of 10 can be a lot, but this can be very helpful for determining if your answer is reasonable and expected. Especially in homework problems, with significant amounts of algebra, first knowing the ballpark of what the solution should be is extremely insightful.

Additionally, order-of-magnitude estimation can be used to determine surprising results that may initially seem like ill-defined or even impossible to solve. As such, they are also often called "Fermi Problems", from Enrico Fermi, a mid-20th century physicist who mastered the art of estimation. Fermi was a scientist on the Manhattan Project and witnessed the first atomic bomb at the Trinity site. Though he and other viewers were a few miles away from the blast, ~~the~~ wind from the shockwave reached the viewers. Fermi tore up a piece of paper and dropped it, in doing so estimating the wind's speed. From the wind speed and the distance from the blast site, Fermi was able to estimate the yield of the bomb; that is, the total energy that it produced. Though his estimation procedure was crude, he was within a factor of 2 of the correct result of the yield.

Another such problem attributed to Fermi was his question of how many piano tuners are there in Chicago? Fermi was a professor at the University of Chicago and a piano player, so this was a relevant problem. However, on its surface it seems wildly ill-defined. The key to this order-of-magnitude estimation is that we be systematic and reasonable with all of our guesses. If we want to determine the number of piano tuners, we need to determine the number of pianos, and the number of people who might own those pianos. So, let's do this estimate.

Chicago has about 10 million people in the metropolitan area. If a house has a piano, it most likely only has a single piano. How many houses are in Chicago? Probably there are 2 people on average in a house, so there are about 5 million houses. It is definitely too much to say that <u>every</u> house has a piano and too little for every 1000th house, but perhaps we could belive one piano per square block. That's about 1 piano per 10 or so houses. With 5 million houses and one piano per block, that's about 500,000 pianos. A piano is tuned how often? Every day is way too often, and every 10 years is much too infrequently. If you play piano regularly, you would probably want it tuned once a year. So, we need 500,000 pianos tuned once a year. How many piano tuners are needed to do this work? Well, how long does it take to tune a piano? Definitely more than 1 minute, but less than a full workday otherwise business would be slow.

So, perhaps it takes about an hour to tune a piano. One tuner could tune 8 pianos in a work day or 40 in one work week. Working 50 weeks a year, a tuner could tune 2000 pianos. If one tuner can tune 2000 pianos in a year, then 250 piano tuners could tune all 500,000 pianos in Chicago. Thus, we estimate that Chicago has 250 piano tuners. In 2009, Chicago had 290 piano tuners. We are amazingly close!

While this is somewhat of a silly exercise but nevertheless exhibits the power of this tool, order-of-magnitude estimation is extremely powerful for the informed citizen. These techniques can be used to determine if a claim in the news is believable or, pardon me, bullshit. A couple years ago, two professors at the University of Washington created a course titled "Calling Bullshit" as an exploration of tools often used in science but applied to all sorts of problems, questions, and claims. I've provided a link to their course website and I recommend checking it out, especially the lecture video on Fermi Estimation (lecture 2.4).

Let's apply this estimation technique to some physics problems you may encounter. As with dimensional analysis, this can be extremely powerful for checking if your answer makes sense at all.

So, here's our question: what is the farthest possible distance that a human can hit a golf ball on Earth?

This problem satisfies the first requirement: it is relatively ill-defined. However, by breaking it apart we can make progress and come to a solution. Let's first consider the relevant quantities we need.

A golf ball is a projectile and the time that it can be in the air depends on the acceleration due to gravity, g. If g is larger, then there is greater acceleration and the ball is in the air for less time (and conversely). This also suggests that if we know the time-of-flight, we can estimate the distance. What's a reasonable time of flight? I'm not sure, but let's do some guesses to see if it makes sense.

Is a time-of-flight of one second reasonable? That seems too short. For order-of-magnitude estimations, we would next guess 10 second time-of-flight. That does sound okay, but just to make sure, let's consider multiplying by another factor of 10. Is a 100 second time-of-flight reasonable? A full minute and a half? That seems exceptionally long (if you don't think so, then tonight sit quietly staring at the wall for 100 seconds). So, we'll estimate a time-of-flight of 10 seconds.

Now, given $g = 10\,m/s^2$ and $t = 10\,s$, how do we make that a distance ~~usig~~ using dimensional analysis? Well, the units of

$d = gt^2$ are a distance in meters,

So we estimate: $d_{max} \sim 10 \cdot 10^2\,m = 1000\,m$.

According to the internets, the longest drive was about 500 yards, or about 500 m. So, we're within a factor of 2 with simple guesses!

Let's attack this problem in a different way. g is always a relevant quantity, but let's instead use the initial ball speed v_0 to estimate the longest drive. With v_0 having dimensions of meters/second, the quantity

$d = \frac{v_0^2}{g}$ has units of length (meters).

So, we just need to estimate v_0 and we can estimate d.

Let's see what makes sense. We'll think about speeds v_0 in miles per hour as that is likely what most of you are more familiar with. At the end, we'll convert to meters/second to get our result. Does $v_0 = 1$ mph make sense? No, this is a slow walking pace. What about 10 mph? Still too slow; this is a brisk jogging pace, but golfballs can go much faster. What about 100 mph? This seems reasonable; the fastest thrown baseballs in the Major Leagues are 100 mph. To see if this makes sense, let's check $v_0 = 1000$ mph. This is the speed of fighter jets, and is significantly above the speed of sound! Do golf balls break the sound barrier? Eh, no, ~~that~~ there aren't sonic booms at the British open. So, 100 mph it is.

$100 \text{ mph} = \frac{100}{2.2} \text{ m/s} \simeq 40 \text{ m/s}$. With this estimate for the golf ball speed, the estimate of the distance traveled is

$$d_{max} \sim \frac{v_0^2}{g} \sim \frac{1600}{10} \text{ m} \sim 160 \text{ m}.$$

This is about a factor of 3 smaller than the longest drive. Again, reasonable, within an order of magnitude, and with a better approximation for v_0, we can get a better estimation for d_{max}.

By the way, the longest golf ball drive ever wasn't on Earth at all. Alan Shepard, an astronaut on Apollo 14 hit three golf balls on the Moon, believed to be the longest drives ever. Can we make sense of this from our calculations above? As we will discuss later in this course, the acceleration due to gravity on the Moon is less than that on Earth, both due to the facts that the moon is smaller than Earth and because it is less dense. Anyway, we have

$$g_{Earth} > g_{Moon}.$$

A golfer on the Moon would swing a club with the same (or approximately the same) velocity as on Earth, as that is determined by the golfer's fitness. So, we expect the ~~ve~~ speed V_0 of the ball to be the same on Earth as on the moon. Then, the distance that said golfer would hit the ball on the moon is

$$d_{moon} \sim \frac{V_0^2}{g_{moon}} > \frac{V_0^2}{g_{Earth}}.$$

In fact, $g_{moon} \sim \frac{g_{Earth}}{6}$, so the golfer could hit the ball about 6 times farther on the moon than on Earth!

That's it for today; don't forget to do homework for Friday and tip your waitstaff; they're working hard for you!