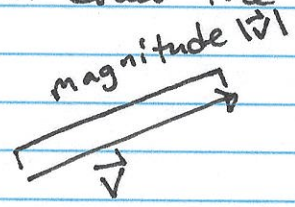
Physics 101 Lecture 5

Welcome to the end of week two! You've made it this far! Turn in Homework please!

We've discussed in some detail Kinematics/motion in one dimension, but this will only get us so far in understanding the phenomena of our universe. In particular, to some approximation, motion of a system may be able to be modeled as one-dimensional, but our universe is three-dimensional so we can do much more than just move along a line. So, our task today and next week is to introduce the language used to describe objects and systems in multiple dimensions. Our fundamental object for doing so will be a vector.

Simply, a vector is a quantity that has a magnitude and definite direction in space. A magnitude is a non-negative number that specifies the length of the vector. The direction of a vector is simply how the vector points with respect to a specified origin. For example, we might draw the vector $\vec{v}$ as:



The length of the arrow is the vector $\vec{v}$'s magnitude, denoted as $|\vec{v}|$. The head of the arrow tells us what direction the vector points, with respect to the origin, which, by convention, is located at the tail of the arrow.

We've already seen vectors in this course before, but not really in that language. Last week we had modeled my strolling across the well. We had defined one point in the well to be the origin, and my displacement from that origin could be plotted as a function of time. This displacement $\vec{d}$ is a vector; it has a magnitude and a direction. The magnitude $|\vec{d}|$ is the distance from the origin; that is the number meters, say, that a measuring device (stick, tape, etc.) would read from me to the origin. We had also discussed a direction: displacement is positive to the right and negative to the left. In this way a displacement $\vec{d}$ of

$\vec{d} = -5\,m$ means or can be read as "five meters to the left of the origin."

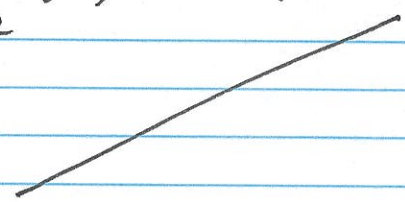
Vectors require a well-orderedness to be well-defined and unambiguous. In the example just discussed, this essentially follows from the well-orderedness of the real numbers. A negative real number means left of the origin (0), while a positive real number means to the right.

Enough about 1D, though. Our universe has three spatial dimensions: left-right, up-down, forward-back. What we mean by a "dimension" can be understood practically as just the number of ways that one can travel through space. Mathematically, a dimension is an <u>independent direction</u> in space. That is, one can move left or right completely independently of up and down. One can move left, say, without ever changing the relative value of "up-ness".

Three dimensions is hard to draw, so let's just work in two-dimensions which will essentially illustrate all of the subtleties of multiple dimensions. This board or paper is two-dimensional: one can move left-right or up-down on the board. This can be done completely independently. For example the line

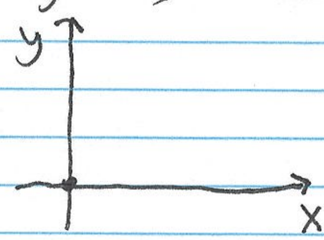
only lives in the left-right dimension. By contrast the line

only exists in the up-down dimension. These two dimensions/lines are independent in the sense of the following. For the left-right line, we can move along the line, changing our left-right location, but without changing the up-down location at all. The following line

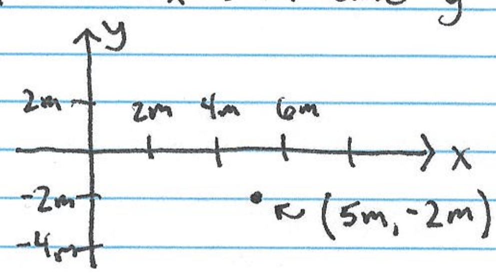


exists in both the left-right and up-down dimensions. Traveling along it changes both the left-right and up-down position.

How can we represent these features? As always, we first need to pick an origin; a point from which everything is measured. We'll denote it by a dot:

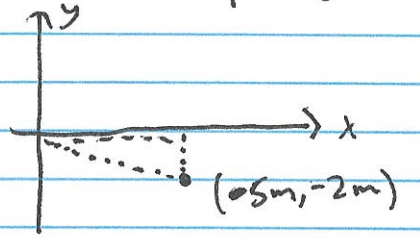


From this dot, we then need to denote our two dimensions. We can do this by drawing axes, both left-right and up-down. The left-right axis is called the abscissa and the up-down axis is the ordinate, but we often colloquially just say "x-axis" and "y-axis", respectively. Now, we can represent any point on the board by it's left-right coordinate ("x component") and up-down coordinate ("y component"). For example, a point with $x = 5\,m$ and $y = -2\,m$ would be somewhere like so:



We can also express this point as an ordered pair of numbers (x_o, y_o), which are the x- and y-coordinates of the point with respect to the origin, respectively.

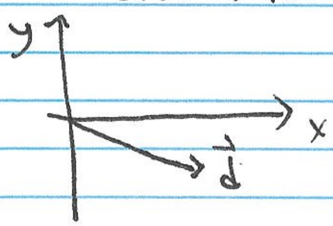
This picture is very powerful. We can immediately determine the distance of this point from the origin. For the example given, we can draw the triangle:



This triangle has sides of length 5m and 2m that meet at a right angle. Therefore, the distance to the origin, d, the hypotenuse of the triangle, is simply the application of the Pythagorean theorem:

$$d = \sqrt{5^2 + 2^2} = \sqrt{25+4} = \sqrt{29}.$$

Now, if this point represented my location traveling in the well, we can define an arrow that points from the origin to my position; my displacement vector, $\vec{d}$. We can draw this as:



This vector $\vec{d}$ can be expressed as

$$\vec{d} = (5m, -2m) = 5m\,\hat{i} - 2m\,\hat{j}.$$

Here, $\hat{i}$ and $\hat{j}$ are called unit vectors and simply represent the direction of the axes in our drawings.

The power of good notation is that it immediately enables an extension of what we've developed here. For example, given my initial position of $\vec{d} = (5m, -2m)$, what is my position vector after walking in the y-direction 10m? ~~Three~~ Four possible answers are:

a) $\vec{d}_{new} = (15m, -2m)$ b) $\vec{d}_{new} = (5m, 8m)$

c) $\vec{d}_{new} = (5m, 2m)$ d) $\vec{d}_{new} = (5m, 10m)$

Talk with your neighbors for a minute!

To solve this problem, we can use vector addition, which exploits the independence of the dimensions. My initial vector is:

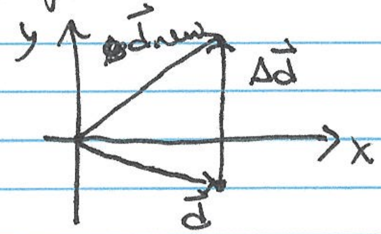
$$\vec{d} = (5m, -2m),$$ as measured from the origin. Now from this point, I move 10 m in the y-direction. That is, if I consider my current location as the origin, then, my displacement from this point $\Delta\vec{d}$ is:

$$\Delta\vec{d} = (0, 10m).$$

Now, here's the beauty of all of this. To find my displacement $\vec{d}_{new}$ from the "true" origin, all I have to do is add $\vec{d}$ to $\Delta\vec{d}$:

$$\vec{d}_{new} = \vec{d} + \Delta\vec{d} = (5+0m, -2+10m) = (5m, 8m).$$

Note that addition proceeds component-wise. This addition has a lovely picture too. First, my current displacment is measured from the origin:



Next, I consider my current position the origin, and then displace by $\Delta\vec{d}$ from it. That is, we draw the vector $\Delta\vec{d}$ starting from the head of $\vec{d}$:

Finally, the total displacement $\vec{d}_{new}$ is the vector formed from connecting the "true" origin to the end of $\Delta\vec{d}$. This "head-to-tail" visual vector addition is extremely powerful and we'll exploit it throughout the semester. By the way, the magnitude of my new displacement is:

$$|\vec{d}_{new}| = \sqrt{5^2+8^2} = \sqrt{25+64} = \sqrt{89}.$$

Sorry, these aren't nice numbers!

A few things to note: independent dimensions represent distinct orthogonal/perpendicular/right directions in which one can travel in space. Our first step in basically every physics problem we will encounter is to identify the origin and set up orthogonal axes, as physics in the different dimensions will largely be independent of one another.

This notation also allows us to express lines or general curves in two-dimensional space. For example, the equation for a line is the standard:

$$y = mx + b.$$

A generic point on the line with x- and y-coordinates $p = (x, y)$ can be expressed as

$$p = (x, mx + b).$$

That is, given a point x, the point y is uniquely determined by the equation for a line. Further, the line is one-dimensional; you only need to specify a single point (x) to know <u>both</u> coordinates.

This can be extended to any curve. For example, a parabola can be expressed as:

$$y = ax^2 + bx + c, \text{ for some real numbers } a, b, c.$$

Does this parabola represent a one-dimensional object? Why or why not? Can you construct curves for which <u>every</u> point in 2D space lives somewhere on the curve?

Don't forget to do homework and have a good weekend!