Physics 342 Lecture 1

Welcome to Physics 342, Introduction to Quantum Mechanics! I'm Andrew Larkoski and I'll be your guide through this fascinating, counter-intuitive, and extremely rich subject. Quantum mechanics underlies nearly all of contemporary physics research; from the inner workings of atoms, to properties of materials, to the physics of neutron stars, to what happens at a black hole. Additionally, quantum mechanics and its consequences are exploited in modern techology and is beginning to produce major breakthroughs in computing. This semester, we will introduce the formalism, axioms, and a new way of thinking quantum mechanically that will require a reinterpretation of the classical physics you have learned.

In this lecture, I will just go through the syllabus, class expectations, and introduce what our over-arching goal this semester is. First on the syllabus: office hours. My office is directly across the hall, in P124 and I will hold office hours:

Monday 2-4 pm, Wednesday 3-4 pm, Thursday 10am-noon

You are <u>required</u> to attend office hours once in the first three weeks of the class, so I can get to know you all, and lower the barrier to asking questions.

The textbook for the course will be my lecture notes which I will provide on my website. Additionally, I recommend David and Darrell's book as a secondary resource. Some homework problems will come from that book and its good to get a second opinion on such an unfamiliar topic. David and Darrell's book is <u>not</u> the primary text for a couple reasons: I think it's introduction of material is out of order, and it does not cover topics that I feel should be in an introductory course.

There will be three graded aspects to this course: weekly homework, a final project, and an oral exam. (I am anti-in class exams because they don't prepare you for anything an actual physicist does.) Homework will cover the topic of that week, and be due in class the following Friday. The final project will be a 5(ish)-page, Physical Review-style paper written up about a relevant topic of interest in quantum mechanics, broadly defined. I will provide a list of possible topics, or you can suggest something you are interested in, with my approval. More details about the final project will be provided after spring break. Finally, there will be oral exams administered during finals week. You will sign up for a 30 minute slot on my door and I will ask you questions about quantum mechanics during that time. The amounts to which each of these components contributes to your final grade is provided on the syllabus. In calculating your grade, I will also drop the lowest homework.

A tentative schedule of topics we will cover is also provided on the syllabus. It may change throughout the semester, but gives you an idea of buzzwords to google to see what we are learning that week. Related to the schedule, I will be away to a conference in Japan the week of 17 February. There will not be lecture at our regular meeting time, as a consequence. However, I will pre-record my lectures for that week and make the videos available to you, in addition to lecture notes. Homework will therefore be assigned and collected as usual that week.

Any questions about the syllabus?

Now, let's get into the topic at hand. First, I want to motivate the structure of topics we will cover in this course. In your first course in (classical) physics, the organization of topics was likely the following:

1) Introduce formalism / language
   Kinematics; acceleration, velocity as concepts
2) Define relevant units / quantities
   SI units: meters, seconds, etc.
3) Identify the fundamental equation
   $\vec{F} = m\vec{a}$
4) <u>Many</u> examples of fundamental equation
   friction, ramps, springs, etc.
5) Conservation laws
   Energy, momentum, angular momentum
6) Weird stuff with rotations / oscillations
   precession, etc.

For each broad topic, I have also provided explicit examples of each from Physics 101, for example. When first introduced to a topic, you need to learn the language in which you express that topic. In classical mechanics that is kinematics, and concepts like acceleration, while in quantum mechanics that will be linear operators, states, and something called the "Hilbert space." This language requires agreed-upon measurement conventions (meters, etc.) and in quantum mechanics we will see that there is a new, fundamental, quantity called "$\hbar$" (read: "h-bar") that sets the scale for everything. With this language established, we then identify the fundamental equation. Classically (at least in Phys 101), this is Newton's second law, while quantum mechanically is will be the Schrödinger equation. The rest of the course is then a study of examples, or reformulation for special systems employing conservation laws: energy, momentum and angular momentum conservation. These conservation laws will be absolutely central to quantum mechanics. Finally, the last week or two in an intro physics course typically focuses on weird rotation phenomena, like precession, and we will see our share of (even weirder!) things near the end of the course. Thus, my goal is to treat this class as if it were ~~a~~ Phys 101, but for a vastly different, and more abstract, topic.

With that outline established, I want to motivate the goals of this course by asking what the goals of physics are. Physics is a science, and as such, knowledge increases through application of the scientific method. That is, we first make a hypothesis for how Nature works. Then we test that hypothesis in an experiment. If the hypothesis agrees with the outcome of experiment, then it lends evidence to the veracity of the hypothesis; importantly, it does not "prove" the hypothesis. (strict logical proof is not possible in empirical science.) If it does not agree, it is discarded, and a new hypothesis is put forth.

This is all well and good, and something you likely learned about in high school, but there was an important temporal phrasing that I used to describe the scientific method. We make a hypothesis for what happens in the future, based on the data we have in the present. That is, the goal of science is to predict the dynamics of some natural system. If the prediction agrees with the experimental data, we claim that we "understand" it.

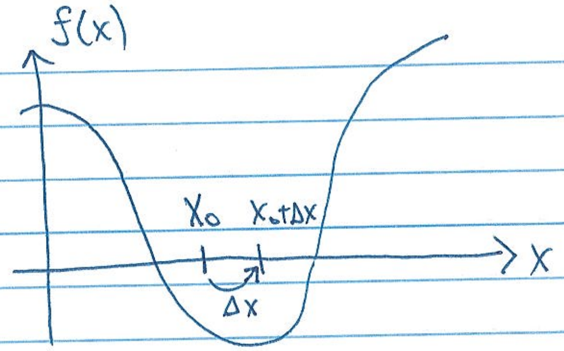
In physics this can be made more mathematically precise. Making a physical prediction means that we determine how a system changes or evolves in time. Any quantity used to describe a system (energy, momentum, position, etc.) has a time derivative if it changes in time. Thus, to make predictions in physics, we want to determine the mechanism responsible for providing a quantity with a time derivative. In classical mechanics, this is very familiar: Newton's second law provides such a mechanism. Newton's second law is:

$$\vec{F} = m\vec{a} = m\frac{d^2\vec{x}}{dt^2} = \frac{d\vec{p}}{dt}.$$

That is, forces are responsible for changing an objects momentum in time. Give me all the forces acting on an object, and Newton's second law tells me how to predict its momentum at any later time. Such a dynamical equation is also called an "equation of motion" and this understanding is why it is the fundamental equation of Phys 101.

So, our ultimate goal in physics, and this class specifically, is to determine that fundamental equation in quantum mechanics and further how systems evolve under that equation. The veracity of the fundamental quantum equation can be tested by comparison of its predictions to experimental data. Spoiler alert: the fundamental equation of quantum mechanics, the Schrödinger equation, has been tested for the past century and every experimental result agrees with its predictions. Which is of course why we teach it!

To end this lecture, and as a teaser for next lecture when we start our quantitative dive into quantum mechanics, I want you to consider a function of one variable; say, $f(x)$. x is a position, for example, and $f(x)$ might be the amplitude of a jiggled rope, say, at position x. Precisely what function $f(x)$ is is not relevant. We can draw an example function as:



On this plot, I have Identified the point x_0, at which the function takes a value $f(x_0)$. Now, from x_0, how do I move along in x to get to a new point a distance Δx to the right? The function value at this new point is, of course, $f(x_0+\Delta x)$. However, to get there from the point x_0, that is, to establish the value $f(x_0+\Delta x)$ exclusively from data at x_0, we can use the Taylor expansion:

$$f(x_0+\Delta x) = f(x_0) + \Delta x \left.\frac{\partial f}{\partial x}\right|_{x=x_0} + \frac{\Delta x^2}{2} \left.\frac{\partial^2 f}{\partial x^2}\right|_{x=x_0} + \ldots$$

(By the way, I will almost always write derivatives in this class as partial derivatives, and not total derivatives d/dx.) To move a distance Δx away from x_0, I need to know <u>all</u> of the derivatives of $f(x)$, evaluated at x_0. This seems very complicated and like we would actually need an infinite amount of data to proceed. However, let's re-write this Taylor expansion in the compact form:

$$f(x_0+\Delta x) = \sum_{n=0}^{\infty} \frac{\Delta x^n}{n!} \left.\frac{\partial^n f}{\partial x^n}\right|_{x=x_0}$$

Further, because the derivative is a <u>linear</u> operator, we can write:

$$f(x_0+\Delta x) = \left(\sum_{n=0}^{\infty} \frac{\Delta x^n}{n!} \frac{\partial^n}{\partial x^n}\right) f(x) \Big|_{x=x_0}$$

Now, the sum in parentheses looks <u>very</u> familiar. If we just think of the multiple derivative

$\frac{\partial^n}{\partial x^n}$ as multiplying $\frac{\partial}{\partial x}$ with itself n-times, the sum has the form of:

$$\sum_{n=0}^{\infty} \frac{a^n}{n!} = e^{a}, \text{ the exponential function!}$$

So, let's just use this generous interpretation for now so that the sum is

$$\sum_{n=0}^{\infty} \frac{\Delta x^n}{n!} \frac{\partial^n}{\partial x^n} = e^{\Delta x \frac{\partial}{\partial x}}$$

What does it mean to "exponentiate" a derivative? More next time...