Parity Violation in the Weak Interaction Lecture 18

Last week, we ended our discussion of QCD with the striking prediction of jets: collimated streams of high energy particles. Jets were discovered in the 1970s and have been extremely important in the further understanding and experimental verification of the Standard Model. In this lecture, as the foundation for the rest of this course, we are going to turn back the clock to the 1930s and 1950s when people were beginning to identify another fundamental force. This will be the weak force, and it will have just as many surprises as the strong force.

In the 1930's, with the discovery of the neutron, it was shortly thereafter realized that the neutron decays. Free neutrons not bound in an atomic nucleus have a half-life of about 15 minutes and are observed to decay to a proton and an electron:

$$n \rightarrow p^+ + e^-$$

This decay is allowed by energy-momentum conservation (the mass of the neutron is 939.6 MeV, the mass of the proton is 938.3 MeV, and the mass of the electron is 511 keV) and by charge conservation, because the neutron is neutral. This model of the decay makes definite predictions: the neutron must be a boson (spin 0 or 1) because the proton and electron are both spin $1/2$ particles, and in an experiment, the energy of the electron is a unique value in the neutron rest frame. In the neutron rest frame, we would see:

$$p^+ \longleftarrow n \longrightarrow e^-$$

We can determine the energy of the elektron via momentum conservation:

$$p_n^2 = (p_p + p_e)^2 \Rightarrow m_n^2 = m_p^2 + m_e^2 + 2\, p_p \cdot p_e$$

Let's take the four-momenta of the proton and electron to be:

$$p_p = \left(\sqrt{m_p^2 + p^2}, 0, 0, p\right), \quad p_e = \left(\sqrt{m_e^2 + p^2}, 0, 0, -p\right)$$

in the neutron rest-frame. Solving for p and plugging in numbers, we expect that the electron in this decay has an energy of $E_e = 1.2$ MeV. Thus, running this experiment over and over, we should repeatedly find electrons with energy of 1.2 MeV.

This experiment was done and was not what was observed. Instead of the electron always having this energy, it was observed to have ~~a~~ distribution of energies, that was always less than 1.2 MeV! Additionally, the prediction that the neutron is a boson is inconsistent with experiments observing radioactive decays. Atoms before and after radioactive decays were observed to still be fermions (= ½-integer spin), which is inconsistent with the spin-0 hypothesis.

These observations led Wolfgang Pauli, Enrico Fermi, and others to postulate the existence of the neutrino, a massless (or, rather, very small mass) spin-½ particle that was also produced in the decay of a neutron:

$$n \rightarrow p^+ + e^- + \bar{\nu}$$

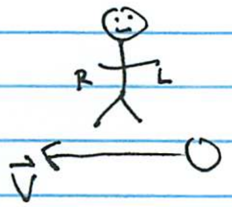
The (anti) neutrino is electrically neutral (neutrino meaning "little neutral one" in Italian).

Fermi introduced a phenomenological model to describe this decay, but it turned out to be incorrect for subtle and fascinating reasons. So, I won't discuss his theory here. Instead, I will work to describe one of the most startling particle physics experiments of the 20th century, led by Chinese-American physisist Chien-Shiung Wu. To describe it, though, we need some background.

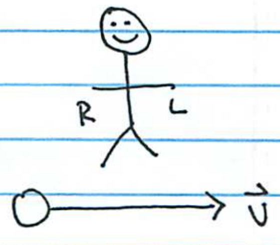
In our discussion of Lorentz transformations long ago, there were classes of possible transformations that we ignored, mostly for simplicity. These were the "large" transformations, that corresponded to matrices with negative determinant. One example of such a transformation is parity; or, the transformation that inverts all spatial axes. The parity operator, P, when acting on the position vector $\vec{x}$ negates it:

$$P\vec{x} = -\vec{x}$$

Parity can be thought of as viewing the physical system in a mirror. Vectors, like position $\vec{x}$, will be flipped in the mirror. For example, imagine that there is a toy car that rolls by to your right:

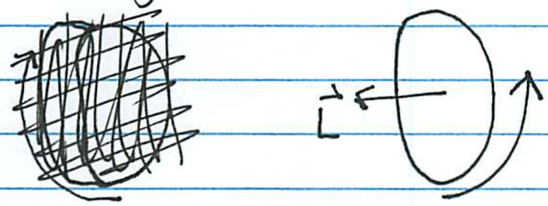


In a mirror, this will look like:



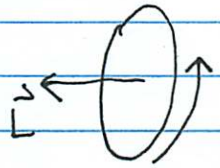
Properly, only objects that flip under parity are vectors. There are objects that do not flip under parity; these are called pseudovectors. Because they do not flip when viewed in a mirror, one could call them "vampire vectors".

An example of a pseudovector is angular momentum. Consider a spinning wheel, that from your perspective is rotating bottom-over-top:



I've denoted the direction of angular momentum also.

In a mirror, you would see the exact same thing:



The reason for this is because angular momentum $\vec{L}$ is formed from the cross-product of two (true) vectors; position and momentum. Angular momentum is:

$\vec{L} = \vec{x} \times \vec{p}$, and under parity: $P\vec{x} = -\vec{x}$ and $P\vec{p} = -\vec{p}$ and

so $P\vec{L} = \vec{L}$. Pseudovectors, like angular momentum, do

not transform under parity. Other examples of pseudovectors are torque or magnetic field. Both are defined from a cross product of vectors, which is why they do not transform.

Note that two applications of parity return to the initial state, and so $P^2 = 1$. That is, the eigenvalues of the parity operator are $+1$ and -1, and this can potentially be a useful quantum number to classify particles. For example, the pions π^+, π^-, and π^0 are pseudoscalars, and transform under parity.

Typically, parity is not so interesting because many physical systems we consider are invariant under it.

For example, Maxwell's equations are:

$$\vec{\nabla}\cdot\vec{E} = \rho \,, \quad \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}, \quad \vec{\nabla}\cdot\vec{B} = 0, \quad \vec{\nabla}\times\vec{B} = \vec{J} + \frac{\partial\vec{E}}{\partial t},$$

of course in natural units. Maxwell's equations are unchanged under a parity transformation:

$$\vec{E}\to-\vec{E}, \quad \vec{\nabla}\to-\vec{\nabla}, \quad \rho\to\rho, \quad \vec{B}\to\vec{B}, \quad \vec{J}\to-\vec{J}.$$

Note that $\vec{E}$, $\vec{\nabla}$, and current density $\vec{J}$ are vectors, while the charge density ρ is a scalar and the magnetic field $\vec{B}$ is a pseudovector. Because Maxwell's equations are unchanged under parity, we say that electromagnetism is parity invariant.

One can also show that the Lagrangian of QCD is invariant to parity transformations. So, the expectation is that fundamental interactions are invariant to parity. More on this in a second.

~~Now~~ Beforewe continue, it is enlightening to introduce two other discrete transformations that can be used to classify elementary physics. One of these is time reversal symmetry, which as its name suggests, flips time:

$$T\,t = -t, \text{ where } t \text{ is time.}$$

Unlike for parity, depending on the vector, it may or may not flip with T. For example, the position $\vec{x}$ is unchanged under T: $T\vec{x}=\vec{x}$, while momentum is flipped:

$$T\vec{p} = -\vec{p}, \text{ because it involves velocity, } \vec{v} = \frac{\Delta\vec{x}}{\Delta t}.$$

Maxwell's equations are invariant under T, as well as P. The T transformations are:

$$\vec{E} \to \vec{E},\ \vec{\nabla} \to \vec{\nabla},\ \rho \to \rho,\ \vec{B} \to -\vec{B},\ \vec{J} \to -\vec{J},\ \frac{\partial}{\partial t} \to -\frac{\partial}{\partial t}.$$

(Why does $\vec{B}$ flip under T?) One can show in a similar way that the QCD Lagrangian is also invariant under T, time-reversal transformations. So, we expect fundamental interactions to be invariant to time reversal.

There is a third discrete transformation that we can perform. It is charge conjugation. This transformation can't easily be thought of as affecting vectors, like position $\vec{x}$ or momentum $\vec{p}$. Its action is to turn particles into anti-particles, and vice-versa, and is denoted by C. Like for P and T, charge conjugation is a symmetry of electromagnetism and QCD (but I won't discuss it more here.).

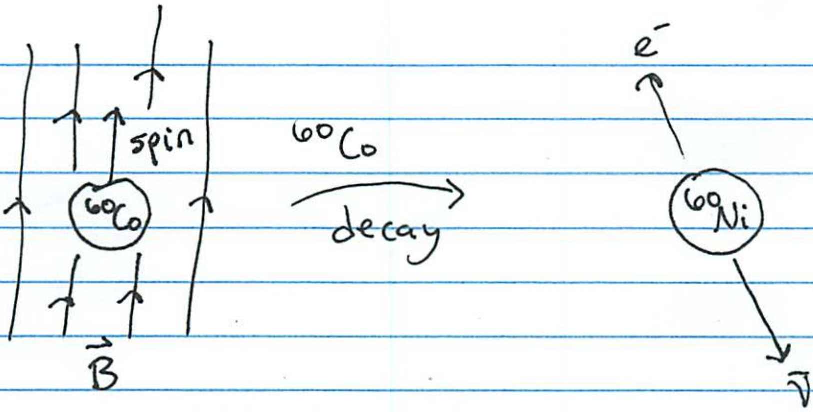
With the definition of C, P, and T, how they transform vectors, and that they are observed to be symmetries of electromagnetism and QCD, one might postulate that they, individually, are symmetries of all possible fundamental interactions. This is reasonable, and most physicists believed it until the 1950's, for reasons we will discuss shortly, Actually, all that is proved regarding C, P, and T is the the application of all of them together is required to be a symmetry, if you have a Hermitian Hamiltonian, and vice versa. This result is known as the CPT theorem, and was rigourously proved by Streater and Wightman in the 1960s. It is entirely possible and allowed for two of C, P, and T to be violated while preserving CPT. Perhaps this would be weird, but nature does not care about our aesthetical tastes.

Now back to where we began this lecture. Neutron decay, or nuclear decay more generally, was an extremely important field of research politically in the 1940s. However, if you just want to harness the energy from nuclear decay, you don't care that much about whether C, P, or T is violated. In the mid 1950s, two physicists named T.D. Lee and C.N. Yang pointed this out, that the force that governs neutron decay may indeed violate parity. This would be weird and unfamiliar because both E+M and QCD preserve parity. Nevertheless, it was not forbidden in neutron decays and there had been no experiments done up to then that could test this.

Enter C.S. Wu. Wu led an experiment at Columbia, where she was a professor, to directly test the parity properties of neutron/nuclear decay. Her experiment was deliciously simple. Wu observed the decay of Cobalt-60 to Nickel-60:

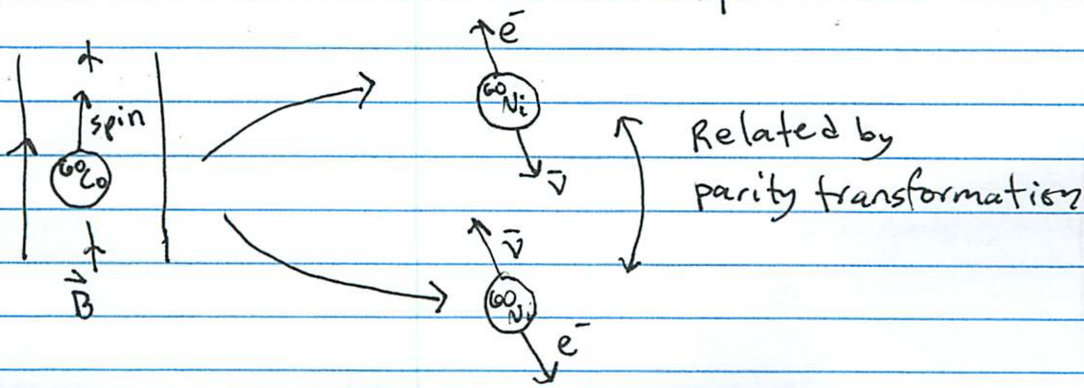
$$^{60}Co \to {}^{60}Ni + e^- + \bar{\nu} + 2\gamma$$

in a magnetic field. The magnetic field was applied to polarize the Cobalt nuclear spin along a preferred axis. Then, one can observe the direction in which the electron is emitted: either parallel or anti-parallel to the direction of ~~the~~ nuclear spin/magnetic field. So, the set up of this experiment is:



The emitted photons are used as a control, and are not essential to the result. How does this test parity violation/conservation? Let's see what the predictions are.

If parity is conserved, then let's imagine what the parity-reversed experiment would be. To determine this, we can just parity transform each component of the experiment. First, the magnetic field $\vec{B}$ does not transform under parity, and neither does the nuclear spin. Both are pseudovectors, as they are defined by a cross-product. On the other hand, the momentum of the electron (or neutrino) are vectors, and as such turn into negative of themselves under parity. That is, under a parity transformation, the experimental set-up is unchanged, (because the important quantities are pseudovectors) while the final configuration is flipped, because you only care about vectors. So, if parity was conserved, then with equal likelihood you should observe the electon parallel to and antiparallel to the direction of nuclear spin:



Then, you watch a bunch of ^{60}Co decays, counting the directions of the electrons, and determine how many were parallel and how many were anti-parallel to the nuclear spin. If parity is not conserved, then you will simply see more electrons in one direction than the other.

In an extremely shocking result, Wu found that more electrons were emitted in the decay opposite to the direction of nuclear spin. When told this, Wolfgang Pauli stated that the result was "total nonsense" and that the experiment "must be repeated." It was, and has been many times since, and the same result was observed. Apparently, the force that governs nuclear decays, in contrast to electromagnetism and QCD, violates parity. We know call this force the weak force, and we will study the properties and consequences of the weak force in the rest of this class.

A few more words on history, then a little more physics. For their observation that the weak nuclear force could violate parity, T.D. Lee and C.N. Yang were awarded the Nobel Prize in 1957, only one year after Wu's experiment. (That's how shocking this was!) C.S. Wu, despite running the experiment that tested parity violation in nuclear decays, was not acknowledged by the Nobel Committee, in what is perhaps the grossest oversight in the history of the Physics prize. Wu was acknowledged with numerous other awards, including the first Wolf Prize in Physics. Wu died in 1997, while, as of these notes, both T.D. Lee and C.N. Yang are still alive. During their study of the weak force, all three were at Columbia.

Now, back to some physics. Let's dive into Wu's observation a bit more. From Stern-Gerlach experiments, we know that the helicity of the electron can be either left or right handed. We also know this from our study of $e^+e^- \to \mu^+\mu^-$ scattering: because muons can have both helicities, this leads to the $1+\cos^2\theta$ distribution of the differential cross section. The brilliance of Wu's experiment is that it provides evidence (though somewhat indirectly) for what the helicity of the unobserved neutrino can be!

That is, one can measure the spin of the nucleus before and after the decay, and the spin of the electron. In these nuclear decays, it is observed that the electron always has left-handed helicity; that is, its direction of spin is always opposite to that of its momentum. Correspondingly, for conservation of anglular momentum, the anti-neutrino in the decay must be right-handed, with its spin parallel to its momentum. Every confirmed experiment of the weak interactions has observed left-handed neutrinos and/or right-handed anti-neutrinos. (Recall that helicity labels of anti-particles are opposite to that of particles.) So, this suggests that only one helicity of neutrinos exists! This is very weird, and unexpected from our experience with electrons.

The fundamental process in the ^{60}Co decay to ^{60}Ni is the decay of a neutron to a proton. In terms of the constituent quarks, this decay is:

$$d \to u + e^- + \bar{\nu}_e$$

where I have added an "e" subscript to the anti-neutrino, as we call it an electron anti-neutrino.

With the spin measurements, in this decay, apparently only left-handed particles know about the weak force. With this observation, then, this decay can be described by the interaction Lagrangian term:

$$\mathcal{L} \supset \frac{4G_F}{\sqrt{2}}\left(\nu_L^\dagger \bar{\sigma}^\mu e_L\right)\left(d_L^\dagger \bar{\sigma}_\mu u_L\right)$$

Here, I have denoted the two-component left-handed spinors of the corresponding fields by their particle name; e_L is the left-handed spinor of the electron, etc. The coefficient G_F is called the Fermi constant, and the 4 and $\sqrt{2}$ are there for historical reasons. The theory in which this interaction was constructed was called the "V-A" theory (vector minus axial) developed shortly ~~before~~ after Wu's experiment. It is the precursor to the weak interactions, which is the fundamental theory. Next lecture, we will discuss the predictions (and shortcomings) of the V-A theory, and the need for a more fundamental description that describes all known phenomena and predicts more.