CP Violation   Lecture 22

In the last two lectures, we resolved the outstanding and exceptionally confusing issue of giving a spin-1 force carrier for the weak force mass. This was acomplished by the Higgs mechanism, where a spin-0 particle, the Higgs, acquires a vacuum expectation value which spontaneously breaks the gauge symmetry and gives the force carrier gauge bosons a mass. Last lecture, we demonstrated that the simplest theory for the unified electro-weak theory is an $SU(2)_W \times U(1)_Y$ gauge theory in which its parameters are overconstrained by experimental measurement. The fact that there are consistent values for the electroweak theory parameters is a huge success, and strong evidence for its description of the weak force.

In this lecture, we will turn our focus from consequences of the electroweak theory from the gauge bosons to consequences for interactions of the fermions. To set the stage for this lecture, let's remind ourselves about what it is we measure to identify particles. This will be of central importance for how we model and understand the properties of fermion interactions with the weak force. In our particle detectors, we are especially sensitive to the energy and momentum of particles, their charge, and, well, that's (typically) about it. So, we must identify particles based on their energy, momentum, and charge. Identifying particles by their charge is what it is: we see, for example, positive or negative charge based on the direction of bending in a magnetic field.

Energy and momentum are a bit subtle, because particles can have in general any energy or momentum. Another way to say this is that the Hermitian operators of energy and momentum, $\hat{H}$ and $\hat{p}$, have a continuous eigenvalue spectrum on the Hilbert space of free particles. In terms of classifying particles, this property isn't useful, because of the continuous infinity of energy and momentum states. However, from $\hat{H}$ and $\hat{p}$, we can form an operator that does yield useful information about the particle. The four-momentum squared operator:

$$\hat{P}_\mu \hat{P}^\mu \equiv \hat{H}\hat{H} - \hat{p}\cdot\hat{p}$$

has a discrete spectrum of eigenvalues that correspond to the squared masses of particles. For example, when acting on the electron wavefunction $|e\rangle$, $\hat{P}_\mu \hat{P}^\mu$ returns:

$$\hat{P}_\mu \hat{P}^\mu |e\rangle = m_e^2 |e\rangle.$$

So, by measuring energy and momentum of particles, we explicitly collapse their wave function into an eigenstate of the squared-momentum operator, or, into a state of definite mass.

This point, that our experiments identify eigenstates of mass, was not discussed when we introduced QCD, nor is it necessary to discuss for electromagnetism. A free particle traveling through space with a definite mass always has a definite mass. It is possible that interactions affect this, but this does not happen in electromagnetism or QCD. An electron in space that emits a photon must still be an electron; its mass did not change. Similarly, an up quark that emits a gluon is still an up quark. It cannot turn into a down quark or a strange quark, or any other quark for that matter. All quarks have different masses and QCD respects this. Another way to express this is that the squared-momentum operator commutes with the flavor operator $\hat{F}$:

$$[\hat{P}_\mu \hat{P}^\mu, \hat{F}] = 0.$$

This means that the eigenstates/particles as identified by their mass is equivalent to particles identified by their type (or interactions with forces). That is, the particle with mass of 511 KeV is the electron flavor, the particle with mass of 106 MeV is the muon flavor, etc. In this way, we say that electromagnetic and QCD interactions are "flavor-diagonal": we can simultaneously diagonalize the squared momentum operator and the flavor operator, if all interactions were only electromagnetism and QCD.

This property is analogous to the central potential problem in quantum mechanics. Particles have energy and z-component of angular momentum, and $\hat{H}$ and $\hat{L}_z$ commute. In quantum mechanics, we can't also label states by $\hat{L}_x$, for example, as $[\hat{L}_z, \hat{L}_x] \neq 0$.

But, as you may be expecting, it didn't have to be this way. It is entirely possible that interactions with a force carrier change the flavor/type of fermion and hence change its mass. In that case then the flavor operator and the squared-momentum operator do not commute, and we are forced to pick a basis in which to define our quantum system. Either states have definite mass, but weird interactions with the force, or they have simple interactions with the force, but ill-defined masses. Because we measure energy and momentum in our experiments, we pick the first choice (for now, anyway...). However, I want to emphasize that this choice is a convention and convenient. If there is a force that is responsible for breaking a flavor symmetry and so

$[\hat{P}_\mu \hat{P}^\mu, \hat{F}] \neq 0$, a measurement of the mass

and flavor of a particle are incompatible quantum mechanically, in general.

If I would have told you this two weeks ago, you might have scoffed because what terribly stupid interaction would do that? However, Nature doesn't care about our aesthetics, and we now know of a force that does just that. The weak force, as mediated by the W boson, turns up quarks into down quarks, or electrons into neutrinos, thereby definitely mixing fermion flavors. Thankfully, weak interactions still do not mix quarks and leptons, so we can study the effects of the non-flavor diagonal weak interactions for leptons and quarks separately. The story with the leptons is much simpler than for the quarks, so let's start with them.

To proceed, we need to write down the coupling of the leptons to the Higgs boson, and then give the Higgs a vev. This will provide the leptons with a mass. The gauge invariant interaction term in the Lagrangian is:

$$\mathcal{L}_{Yuk} = -y_e^{ij} L_a^{\dagger i} \varphi_a e_R^j - (y_e^{ji})^* e_R^{\dagger i} \varphi_a^* L_a^j$$

Here, "Yuk" denotes the Yukawa interaction that corresponds to scalar-fermion coupling. L_a^i is the left-handed lepton doublet that consists of charged leptons and neutrinos:

$$L_a^i = \begin{pmatrix} \begin{pmatrix} \nu_{eL} \\ e_L^- \end{pmatrix}_a \\ \begin{pmatrix} \nu_{\mu L} \\ \mu_L^- \end{pmatrix}_a \\ \begin{pmatrix} \nu_{\tau L} \\ \tau_L^- \end{pmatrix}_a \end{pmatrix}_i$$

Here, the index a is the $SU(2)_w$ index, which determines the coupling of the leptons to the Higgs boson φ, while i is the flavor index that defines which type of lepton it is: either electron, muon, or tau. Note also that these are exclusively left-handed fermions, as neutrinos in the Standard Model only have left-handed helicity. The right-handed leptons, e_R^j, are only those that are charged:

$$e_R^j = \begin{pmatrix} e_R^- \\ \mu_R^- \\ \tau_R^- \end{pmatrix}_j .$$

It is informative to multiply these terms out to see what the structure imposed by this interaction is. Let's just consider the Higgs field set to its vev:

$$\varphi_a = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} .$$

We then have:

$$\mathcal{L}_m = -\begin{pmatrix} e_L^\dagger & \mu_L^\dagger & \tau_L^\dagger \end{pmatrix} \frac{v}{\sqrt{2}} Y_e \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} + \text{h.c.}$$

Here, h.c. denotes Hermitian conjugation of the first term. Y_e is the Yukawa coupling matrix with entries y_e^{ij}. Note that the neutrinos appear no where in this expression. This will imply that they are massless. As we discussed earlier, because of the way in which we do experiments in particle physics, we measure particles with definite mass. For an arbitrary Yukawa matrix Y_e, we can rotate it on the left and right by unitary matrices to bring it to a diagonal form. That is, we can write:

$$\mathcal{L}_m = -\left(e_L^\dagger \; \mu_L^\dagger \; \tau_L^\dagger\right) U_L^\dagger U_L Y_e \frac{v}{\sqrt{2}} U_R^\dagger U_R \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} + \text{h.c.}$$

The matrices U_L and U_R are unitary, and act to rotate the different charged leptons into one another: $U_L^\dagger U_L = \mathbb{1} = U_R^\dagger U_R$. We choos U_L and U_R such that $U_L Y_e U_R^\dagger$ is diagonal:

$$U_L Y_e U_R^\dagger = \begin{pmatrix} y_e & & \\ & y_\mu & \\ & & y_\tau \end{pmatrix}.$$ Then, we just

define the "new" flavors of charged leptons to be

$$\left(e_L^\dagger \; \mu_L^\dagger \; \tau_L^\dagger\right) \to \left(e_L^\dagger \; \mu_L^\dagger \; \tau_L^\dagger\right) U_L^\dagger \quad \text{and}$$

$$\begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \to U_R \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}$$ This then accomplishes the goal of expressing the

masses of the leptons in terms of the eigenstates of the mass matrix (= squared-momentum matrix). The Lagrangian becomes:

$$\mathcal{L}_m = -\frac{v y_e}{\sqrt{2}} e_L^\dagger e_R - \frac{v y_\mu}{\sqrt{2}} \mu_L^\dagger \mu_R - \frac{v y_\tau}{\sqrt{2}} \tau_L^\dagger \tau_R + h.c.$$

In this form we immediately identify, for example, the mass of the electron as:

$$m_e = \frac{v y_e}{\sqrt{2}}.$$

No neutrinos appear here, and so they are massless.

With this choice of basis so that the mass matrix is diagonal, how does this affect interactions with gauge fields? Consider the interactions with the W boson, for example. The W boson couples to the leptons as:

$$\mathcal{L}_W \propto \left(e_L^\dagger \;\; \mu_L^\dagger \;\; \tau_L^\dagger\right) \bar{\sigma}\cdot W^+ \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} + h.c.$$

Because there is no mass for the neutrinos, we can rotate them into one another as convenient. Under the rotation that diagonalized the mass matrix of the leptons, ~~the~~ this interaction becomes:

$$\mathcal{L}_W \to \left(e_L^\dagger \;\; \mu_L^\dagger \;\; \tau_L^\dagger\right) U_L^\dagger \bar{\sigma}\cdot W^+ \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}.$$

So, for simplicity, we can choose to rotate the neutrinos as:

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} \to U_L \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

, and doing so leaves the coupling to the W boson diagonal!

That is:

$$\mathcal{L}_W \to \left(e_L^\dagger \;\; \mu_L^\dagger \;\; \tau_L^\dagger\right) U_L^\dagger \bar{\sigma}\cdot W^+ U_L \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} = \left(e_L^\dagger \;\; \mu_L^\dagger \;\; \tau_L^\dagger\right) \bar{\sigma}\cdot W^+ \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}.$$

~~So,~~ ~~The~~ This argument can be extended to all gauge interactions of the leptons. So, precisely because neutrinos are massless (we can rotate them into one another without affecting the structure of masses) we can simultaneously diagonalize the squared-momentum operator (= mass) and the flavor (= gauge interactions), for the leptons. We'll have to correct the masslessness of neutrinos next lecture, but for now, it is an extremely good approximation.

So, what if we do the same exercise for the quarks? Now, the Yukawa interactions for the quarks are:

$$\mathcal{L}_{Yuk} = -y_d^{ij} Q_a^{i\dagger} \varphi_a d_R^j - y_u^{ij} Q_a^{i\dagger} \varepsilon_{ab} \varphi_b^* u_R^j + h.c.$$

Unlike for the leptons, both up and down quarks have a mass, which is why there are right-handed up and down quarks. The left-handed quark field Q_a^i is:

$$Q_a^i = \begin{pmatrix} \begin{pmatrix} u_L \\ d_L \end{pmatrix}_a \\ \begin{pmatrix} c_L \\ s_L \end{pmatrix}_a \\ \begin{pmatrix} t_L \\ b_L \end{pmatrix}_a \end{pmatrix}_i$$

while the right-handed fields are:

$$u_R^i = \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix}_i, \quad d_R^i = \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix}_i$$

, which ranges over the three generations of quarks.

Plugging these into the Yukawa interaction with the Higgs boson at its vev, we have:

$$\mathcal{L}_m = -\begin{pmatrix} u_L^\dagger & c_L^\dagger & t_L^\dagger \end{pmatrix} \frac{v}{\sqrt{2}} Y_u \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix}$$

$$-\begin{pmatrix} d_L^\dagger & s_L^\dagger & b_L^\dagger \end{pmatrix} \frac{v}{\sqrt{2}} Y_d \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} + h.c.$$

Note what is different with the lepton case! Now there are <u>two</u> Yukawa matrices, one for up-type quarks, one for down type quarks. As with leptons, we can rotate the mass matrix so that the particles (the quarks) are eigenstates of the squared-momentum matrix, as we measure. The mass terms of the Lagrangian are then:

$$\mathcal{L}_m = -\begin{pmatrix} u_L^\dagger & c_L^\dagger & t_L^\dagger \end{pmatrix} U_{uL}^\dagger U_{uL} \frac{v}{\sqrt{2}} Y_u U_{uR}^\dagger U_{uR} \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix}$$

$$- \begin{pmatrix} d_L^\dagger & s_L^\dagger & b_L^\dagger \end{pmatrix} U_{dL}^\dagger U_{dL} \frac{v}{\sqrt{2}} Y_d U_{dR}^\dagger U_{dR} \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} + h.c.,$$

where we choose matrices U_{uL}, U_{uR}, U_{dL} and U_{dR} such that:

$$U_{uL} Y_u U_{uR}^\dagger = \begin{pmatrix} y_u & & \\ & y_c & \\ & & y_t \end{pmatrix}, \quad U_{dL} Y_d U_{dR}^\dagger = \begin{pmatrix} y_d & & \\ & y_s & \\ & & y_b \end{pmatrix}.$$

By defining the new flavors of leptons to be rotated by the unitary matrices, the mass termof the Lagrangian is flavor diagonal:

$$\mathcal{L}_m \to -\begin{pmatrix} u_L^\dagger & c_L^\dagger & t_L^\dagger \end{pmatrix} \frac{v}{\sqrt{2}} \begin{pmatrix} y_u & & \\ & y_c & \\ & & y_t \end{pmatrix} \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix}$$

$$- \begin{pmatrix} d_L^\dagger & s_L^\dagger & b_L^\dagger \end{pmatrix} \frac{v}{\sqrt{2}} \begin{pmatrix} y_d & & \\ & y_s & \\ & & y_b \end{pmatrix} \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix}. \text{ Cool!}$$

But we're not out of the woods yet. What happens to the interactions with the W boson? For the quarks, the interaction with the W boson proceeds as:

$$\mathcal{L}_W \propto \begin{pmatrix} u_L^\dagger & c_L^\dagger & t_L^\dagger \end{pmatrix} \bar{\sigma}\cdot W^+ \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + h.c.$$

To make the mass terms of the Lagrangian diagonal, we had to multiply by the unitary matrices that rotated the quark basis. We have to do the same thing here. Unlike for the leptons, we have no freedom in rotating the quarks. Now, the coupling to the W boson after rotation to the mass eigenbasis is:

$$\mathcal{L}_W \to \begin{pmatrix} u_L^\dagger & c_L^\dagger & t_L^\dagger \end{pmatrix} U_{uL}^\dagger U_{dL}\, \bar{\sigma}\cdot W^+ \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$$

In general, $U_{uL}^\dagger U_{dL} \neq \mathbb{1}$, and so quark flavors are mixed by the weak interactions! (One can show that the photon + Z boson cannot mix flavors). So, apparently, for the quarks the squared momentum operator does not commute with the flavor operator:

$[\hat{P}_\mu \hat{P}^\mu, \hat{F}] \neq 0$, and so the mass matrix and weak interactions can not be simultaneously diagonalized. We call the mixing matrix

$$U_{uL}^\dagger U_{dL} = V_{CKM}, \text{ the}$$

Cabbibo-Kobayashi-Maskawa matrix (CKM matrix).

We're not out of the woods with weirdness yet. Hidden in the CKM matrix is a dark and dirty secret: particles and anti-particles do not have the same couplings to the W bosons! Let's see what this means. Let's first consider the simpler case of just two generations of quarks. In this simpler case the interactions with the W boson are:

$$\mathcal{L}_W^{2G} = \begin{pmatrix} u_L^\dagger & c_L^\dagger \end{pmatrix} V_{CKM}\, \bar{\sigma}\cdot W^+ \begin{pmatrix} d_L \\ s_L \end{pmatrix} + \begin{pmatrix} d_L^\dagger & s_L^\dagger \end{pmatrix} V_{CKM}^\dagger\, \bar{\sigma}\cdot W^- \begin{pmatrix} u_L \\ c_L \end{pmatrix}$$

The CKM matrix is unitary, as it is the product of two unitary matrices. We therefore can write

$$V_{CKM} = \begin{pmatrix} a e^{i\alpha} & b e^{i\beta} \\ c e^{i\gamma} & d e^{i\delta} \end{pmatrix}$$, for real + positive a, b, c, d and real $\alpha, \beta, \gamma, \delta$.

Imposing the <u>unitarity constraint</u>, we have:

$$V_{CKM}^\dagger V_{CKM} = \mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a e^{-i\alpha} & c e^{-i\gamma} \\ b e^{-i\beta} & d e^{-i\delta} \end{pmatrix} \begin{pmatrix} a e^{i\alpha} & b e^{i\beta} \\ c e^{i\gamma} & d e^{i\delta} \end{pmatrix}$$

$$= \begin{pmatrix} a^2 + c^2 & ab e^{-i(\alpha-\beta)} + cd e^{-i(\gamma-\delta)} \\ ab e^{i(\alpha-\beta)} + cd e^{i(\gamma-\delta)} & b^2 + d^2 \end{pmatrix}$$

Then, $a^2 + c^2 = 1$, $b^2 + d^2 = 1$, <u>$ab = cd$</u>, $\alpha - \beta - \gamma + \delta = \pi$.

That is, we can express the CKM matrix with two generations purely with a magnitude a and three phases, α, γ, and δ. Using the expressions and constraints above, we have:

$$V_{CKM} = \begin{pmatrix} a e^{i\alpha} & -\sqrt{1-a^2}\, e^{i(\alpha-\gamma+\delta)} \\ \sqrt{1-a^2}\, e^{i\gamma} & a e^{i\delta} \end{pmatrix}$$

This CKM matrix has three non-zero phases in general, which means that $V_{CKM} \neq V_{CKM}^\dagger$. As such up quarks and anti-up quarks interact with W bosons differently! However, there's a catch. This matrix has 3 phases and yet mixes 4 fermions (u_L, d_L, s_L, c_L). So, I can perform yet another change to the fields to remove these phases completely! Recall that the phase of a wavefunction is unphysical; it does not affect probabilities. So, I can multiply my fermions by an appropriate phase to completely remove the complex numbers from the CKM matrix!

Multiplying out the W-boson interactions, we find:

$$\mathcal{L}_W^{2G} = a e^{i\alpha} u_L^\dagger d_L + \sqrt{1-a^2}\, e^{i\gamma} c_L^\dagger d_L - \sqrt{1-a^2}\, e^{i(\alpha-\gamma+\delta)} u_L^\dagger s_L + a e^{i\delta} c_L^\dagger s_L + \text{h.c.}$$

If we then rescale the fields as:

$$u_L^\dagger \to u_L^\dagger,\ d_L \to e^{-i\alpha} d_L,\ c_L^\dagger \to e^{i(\alpha-\gamma)} c_L^\dagger,\ s_L \to e^{i(\gamma-\alpha-\delta)} s_L$$

~~So~~ all phases are removed! The Lagrangian becomes:

$$\mathcal{L}_W^{2G} \propto (u_L^\dagger\ c_L^\dagger) \begin{pmatrix} a & -\sqrt{1-a^2} \\ \sqrt{1-a^2} & a \end{pmatrix} \bar{\sigma}\cdot W^+ \begin{pmatrix} d_L \\ s_L \end{pmatrix} + \text{h.c.}$$

$$= (u_L^\dagger\ c_L^\dagger) \begin{pmatrix} \cos\theta_c & -\sin\theta_c \\ \sin\theta_c & \cos\theta_c \end{pmatrix} \bar{\sigma}\cdot W^+ \begin{pmatrix} d_L \\ s_L \end{pmatrix} + \text{h.c.}$$

That is, by an appropriate change of phase to the fields, the two-generation Yukawa matrix is real! The entries of the matrix are typically defined by the Cabbibo angle, θ_c. Note importantly that we could remove the three phases because we had 4 fields: there is an overall phase of the Lagrangian that must be 0 for it to be real. With this adjustment, particles and antiparticles have the identical couplings to the W boson.

Our universe, however, has not two but three generations of quarks. If we go through the same exercise for three generations as for two, you find that the CKM matrix depends on three angles (the Euler angles) and 6 phases. With three generations there are 6 quarks and so, with the constraint that the phase of the Lagrangian is fixed, we can remove 5 of these phases. The three generation CKM matrix is still, irreducibly, a complex matrix!!!

Because of this fact, quarks and anti-quarks in general couple to the W boson differently. So, the transformation of turning quarks to anti-quarks is <u>not</u> a symmetry of nature, even though we started with a system that was completely symmetric. Recall that in a Feynman diagram that particles are turned into anti-particles by flipping the direction of time; therefore the complex phase of the CKM matrix violates T. As CPT must be conserved by the Hermitivity of the Hamiltonian, we equivalently say that the CKM matrix violates CP.

So, the weak interactions just keep giving. Not only is it enough to violate parity with purely left handed interactions, they also violate CP because all quarks are massive <u>and</u> there are three generations.