Neutrino Oscillations Lecture 23

Last lecture, we discussed the exceptionally weird property of the weak interactions to mix different mass eigenstates of quarks. We found that the mass operator (from the measurement of energy and momentum) could not be simultaneously diagonalized with the flavor operator (defined by the interactions of quarks with W bosons). Additionally, because there are three generations of quarks in the Standard Model, this mixing of quarks actually introduces CP violation: quarks and anti-quarks interact differently with the W boson.

The story with leptons, on the other hand, was actually quite simple. Unlike quarks, leptons do not mix into one another under the weak interactions. This argument requires neutrinos to be massless; in that case, we are allowed to arbitrarily rotate the neutrinos into one another to ~~diagonalize~~ diagonalize both the mass matrix and the weak-interaction flavor matrix. The mass matrix for neutrinos in the standard model is the 0 matrix, and multiplying by any unitary matrix keeps the masses 0. So this is the story in the Standard Model; is it true? That is, can we test this prediction?

For simplicity in most of this lecture, we will assume that there are just two different neutrinos. This will capture the majority of the interesting physics, and the extension to the case of three neutrinos (that is, three generations of leptons) is straightforward. So, the prediction in the Standard Model is that neutrinos are massless, and therefore the leptons do not mix under interactions with the weak force. If the leptons do not mix, then the type of neutrino that we produce somehow (in radioactive decay, for example) will be the type of neutrino that we observe always. So, let's see if this is the case.

To do this, let's consider the concrete example of production of neutrinos at a nuclear reactor, say, right here at Reed. The heart of a nuclear reaction, whether it be nuclear fission or fusion, is the decay properties of the neutron:

$$n \to p^+ + e^- + \bar{\nu}_e$$

By energy and momentum conservation, the only possible decay of the neutron is to a proton, electron, and electron anti-neutrino. Note, importantly, that this interaction occurs via the weak force, specifically through an intermediate W boson, and so the decay products are eigenstates of flavor (eigenstates of their interaction with the weak force). This is what enables us to state with certainty that the anti-neutrino is electron-type.

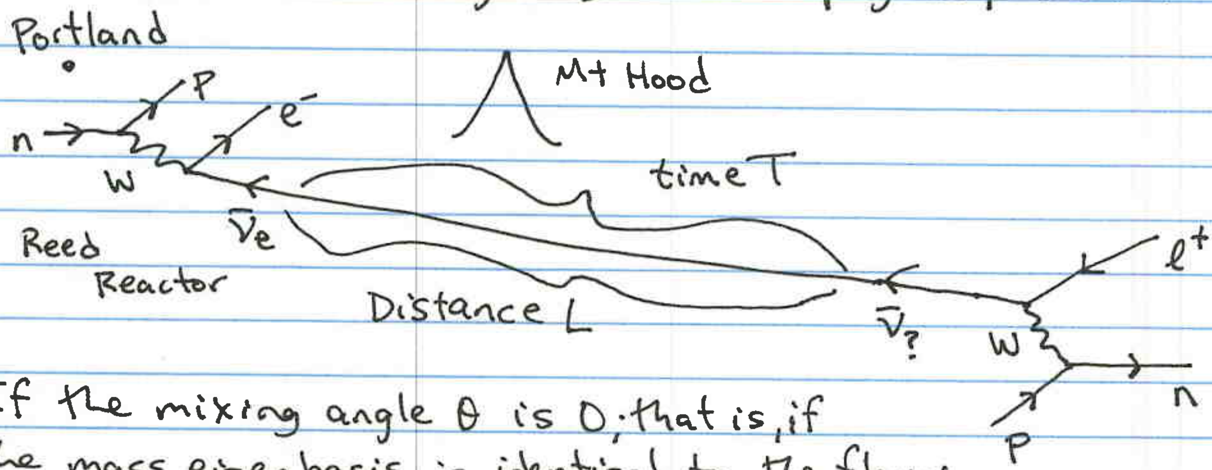
To see if the electron anti-neutrino mixes with other neutrino flavors, let's put a neutrino detector some distance away from the reactor; say, somewhere in Eastern Oregon. How do we measure a neutrino? Essentially exactly opposite to how it was created. We discussed how this is done at the IceCube experiment in homework 10. A neutrino that was traveling along strikes a proton in a target and produces a lepton and a neutron. Because this interaction of the neutrino and proton proceeds via the weak interaction, the flavor of the lepton produced must be the same as the initial neutrino (because the W boson couples to eigenstates of lepton flavor). So, we have an experiment that can test if the neutrinos (and therefore the leptons) mix under the weak interactions. We produce electron anti-neutrinos in the Reed reactor, let them travel to Eastern Oregon, and then see what flavor of charged lepton is produced in the neutrino-proton scattering. Identifying the charged lepton, especially if it is an electron or muon, is pretty easy, so this experiment isn't too challenging, other than the fact that neutrinos interact very, very, very weakly with other matter!

Let's study this experiment and determine the probability to observe an electron neutrino in Eastern oregon. As mentioned earlier, we assume that there are only two flavors of neutrinos, electron- and muon-type. These are the eigenstates of flavor (the weak interactions) while we will call the eigenstates of mass neutrinos 1 and 2. The flavor and mass bases are related by a rotation matrix:

$$\begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}$$

Here, the mixing matrix is defined by some mixing angle θ which is analogous to the case that we studied last lecture with the quarks. In general, this mixing matrix would be a unitary 2x2 matrix, but, as we did with the quarks, we can remove all complex phases by appropriate re-definitions. The neutrino types are labelled by their ~~appropriate~~ respective subscript.

So, with this set up, we want to see how this mixing affects the probability for an electron neutrino to be observed far from its source. A picture of the experiment, with artistic license regarding the non-physics pieses is:



If the mixing angle θ is 0, that is, if the mass eigenbasis is identical to the flavor basis, then we will always observe an electron neutrino in Eastern Oregon.

From our mixing formula, the electron (anti-) neutrino is the ~~two~~ linear combination of the mass basis:

$$|\bar{\nu}_e\rangle = \cos\theta|\bar{\nu}_1\rangle + \sin\theta|\bar{\nu}_2\rangle.$$

As fermions, neutrinos ν_1 and ν_2 satisfy the Dirac equation, and therefore propagate according to the Klein-Gordon Equation. As they are mass eigenstates (eigenstates of the squared-momentum operator), their respective Klein-Gordon equations are:

$$(\partial_\mu\partial^\mu + m_1^2)\psi_{\bar{\nu}_1} = 0, \quad (\partial_\mu\partial^\mu + m_2^2)\psi_{\bar{\nu}_2} = 0,$$

where $\psi_{\bar{\nu}_1}$ and $\psi_{\bar{\nu}_2}$ are the wavefunctions for anti-neutrinos 1 and 2, respectively. These wavefunctions therefore propagate (have position and time dependence of) as:

$$\psi_{\bar{\nu}_1} \propto e^{-ip_1 \cdot x}, \quad \psi_{\bar{\nu}_2} \propto e^{-ip_2 \cdot x}, \text{ where}$$

$$p_1 = (E_1, \vec{p}_1), \quad p_2 = (E_2, \vec{p}_2), \text{ and } x = (t, \vec{x}).$$

Note that we assume that the neutrinos have non-zero masses m_1 and m_2, in general. This is not allowed in the Standard Model, but we want to test this.

Therefore, the general time evolution of the electron anti-neutrino is:

$$|\bar{\nu}_e(t)\rangle = \cos\theta \, e^{-ip_1 \cdot x} |\bar{\nu}_1\rangle + \sin\theta \, e^{-ip_2 \cdot x} |\bar{\nu}_2\rangle.$$

Importantly, note that because $\bar{\nu}_1$ and $\bar{\nu}_2$ have definite masses m_1 and m_2, they each propagate with a definite velocity. By contrast, because $\bar{\nu}_e$ does not have a definite mass, it does not propagate with a definite velocity.

At time $t=0$ and $\vec{x} = \vec{0}$, the electron anti-neutrino is located at the Reed Reactor and is decomposed as:

$$|\bar{\nu}_e(t=0)\rangle = \cos\theta |\bar{\nu}_1\rangle + \sin\theta |\bar{\nu}_2\rangle$$

At time $t=T$ and $|\vec{x}| = L$, the electron anti-neutrino is located at our detector in eastern Oregon. This electron anti-neutrino is decomposed as:

$$|\bar{\nu}_e(t=T)\rangle = \cos\theta \, e^{-i(ET - p_1 L)} |\bar{\nu}_1\rangle + \sin\theta \, e^{-i(ET - p_2 L)} |\bar{\nu}_2\rangle$$

Note a few things about this expression. First, I have assumed that the energy of $\bar{\nu}_1$ and $\bar{\nu}_2$ are the same. This is required for mass basis neutrinos $\bar{\nu}_1$ and $\bar{\nu}_2$ are coherent and construct $\bar{\nu}_e$. (Nota Bene: Griffiths' textbook assumes that the momentum of $\bar{\nu}_1$ and $\bar{\nu}_2$ are the same. This is wrong, but happens to give the correct answer. Nevertheless, proceed there with caution.) Because $\bar{\nu}_1$ and $\bar{\nu}_2$ have different masses, they then have different momenta, p_1 and p_2.

We will make some approximations to simplify the time evolved electron anti-neutrino. First, note that the $\bar{\nu}_e$ from neutron decay has an energy on the order of 1 MeV, corresponding to the mass difference between the neutron and proton. The mass of the neutrino must be tiny, and so it will essentially be traveling at the speed of light. Therefore, in natural units, we will assume that $T = L$, which would be true for light.

We also need to evaluate the difference between energy and momentum, $E - p_i$. Demanding that the anti-neutrino $\bar{\nu}_i$ is on-shell, we have

$$E - p_i = E - \sqrt{E^2 - m_i^2} = E\left(1 - \sqrt{1 - \frac{m_i^2}{E^2}}\right) = E - E + \frac{m_i^2}{2E} + \dots,$$

where we have expanded in the highly-relativistic limit where $m_i \ll E$. Therefore:

$$E - p_i = \frac{m_i^2}{2E} + \dots$$ and so the time evolved

electron anti-neutrino is:

$$|\bar{\nu}_e(t=T)\rangle = \cos\theta\, e^{-im_1^2\frac{L}{2E}}|\bar{\nu}_1\rangle + \sin\theta\, e^{-im_2^2\frac{L}{2E}}|\bar{\nu}_2\rangle.$$

Okay, this is the neutrino that makes it to our detector. Our detector works by identifying the flavor of charged lepton produced in scattering of the anti-neutrino and a proton. So, while mass eigenstates have simple propagation properties, we need to re-express the neutrino at time $t=T$ in terms of its flavor/weak eigenstates to determine how it interacts with our detector.

From earlier, we have:

$$|\bar{\nu}_1\rangle = \cos\theta|\bar{\nu}_e\rangle - \sin\theta|\bar{\nu}_\mu\rangle, \quad |\bar{\nu}_2\rangle = \sin\theta|\bar{\nu}_e\rangle + \cos\theta|\bar{\nu}_\mu\rangle.$$

Then, in terms of $\bar{\nu}_e$ and ν_μ states the neutrino at our detector is:

$$\begin{aligned}
|\bar{\nu}_e(t=T)\rangle &= e^{-im_1^2\frac{L}{2E}}\cos^2\theta|\bar{\nu}_e\rangle + e^{-im_2^2\frac{L}{2E}}\sin^2\theta|\bar{\nu}_e\rangle \\
&\quad - e^{-im_1^2\frac{L}{2E}}\cos\theta\sin\theta|\nu_\mu\rangle + e^{-im_2^2\frac{L}{2E}}\cos\theta\sin\theta|\nu_\mu\rangle \\
&= e^{-im_1^2\frac{L}{2E}}\Big[\Big(\cos^2\theta + e^{i(m_1^2-m_2^2)\frac{L}{2E}}\sin^2\theta\Big)|\bar{\nu}_e\rangle \\
&\qquad - \cos\theta\sin\theta\Big(1 - e^{i(m_1^2-m_2^2)\frac{L}{2E}}\Big)|\bar{\nu}_\mu\rangle\Big]
\end{aligned}$$

Then, the amplitude for an electron anti-neutrino to be produced at Reed and observed as an electron anti-neutrino or a muon neutrino in our detector in Baker City is given by the wavefunction overlaps (ignoring overall phases):

$$\mathcal{M}(\bar{\nu}_e \to \bar{\nu}_e) = \langle \bar{\nu}_e(T) | \bar{\nu}_e(0) \rangle = \cos^2\theta + e^{-i(m_1^2 - m_2^2)\frac{L}{2E}} \sin^2\theta$$

$$\mathcal{M}(\bar{\nu}_e \to \bar{\nu}_\mu) = \langle \bar{\nu}_\mu(T) | \bar{\nu}_e(0) \rangle = \cos\theta \sin\theta \left(1 - e^{-i(m_1^2 - m_2^2)\frac{L}{2E}}\right).$$

The corresponding probabilities are the absolute squares of this:

$$P(\bar{\nu}_e \to \bar{\nu}_e) = |\mathcal{M}(\bar{\nu}_e \to \bar{\nu}_e)|^2 = 1 - \sin^2(2\theta)\sin^2\left((m_1^2 - m_2^2)\frac{L}{4E}\right)$$

$$P(\bar{\nu}_e \to \bar{\nu}_\mu) = |\mathcal{M}(\bar{\nu}_e \to \bar{\nu}_\mu)|^2 = \sin^2(2\theta)\sin^2\left((m_1^2 - m_2^2)\frac{L}{4E}\right)$$

To write these expressions, I used double-and half-angle trigonometric identities liberally. Note, importantly, that the sum of probabilities is 1; that is, the electron anti-neutrino turns into <u>something</u> in its travel across the Cascades.

These expressions are exceptionally interesting. Note that the neutrinos only mix if they are massive, which we already knew from our previous study. However, even more interesting is the fact that they only mix if they have different masses! That is, if $\bar{\nu}_1$ and $\bar{\nu}_2$ had the same mass $m = m_1 = m_2$, we could diagonalize the flavor and mass operators simultaneously. (Can you convince yourself of this?)

I also emphasized that our detector is somewhere in Eastern Oregon. Why is this reasonable? Or, rather, what range of mass-squared differences does this make us sensitive to? Plugging back in the c's and ħ's, the argument of the $\sin^2$ is:

$$\sin^2\left((m_1^2-m_2^2)\frac{L}{4E}\right) = \sin^2\left(1.27\,(m_1^2-m_2^2)(eV)^2\,\frac{L(km)}{E(GeV)}\right)$$

Here, the dimensions have been removed, and we evaluate the distance L in kilometers, the energy of the neutrino E in GeV, and measure the mass splitting in $(eV)^2$. So, for neutrinos produced in neutron decay, we should have a separation of the Reed Reactor from our detector on the order of hundreds of kilometers! That is, to measure eV neutrino masses, we should indeed have the detector in Harney County.

Some other things to note: measuring neutrino oscillations is only sensitive to the squared mass difference. It says nothing about the absolute mass scale; that is, what m_1 is. So, all we can tell from neutrino oscillation is that at least one neutrino has mass that is not zero, and different from the masses of the other neutrinos.

This analysis contains most of the physics of neutrino oscillation, but can be extended to the general case of arbitrary numbers of neutrinos. In particular, the Standard Model has three neutrinos which in general will mix if they have different masses. As with the CKM matrix that defined the mixings of the quarks, there is a 3x3 matrix that describes the mixing of the neutrinos. It is called the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix and is parametrized by three angles (the Euler angles) and one complex phase. Like for the quarks, the existence of a complex phase means that, generically, neutrino oscillations of three generations violates CP. However, unlike the case for quarks, the complex phase of the PMNS matrix has not been measured, so it is not known with certainty what the CP violation of the neutrinos is.

Neutrino oscillations are an unambiguous deviation of observation from Standard Model prediction. The Standard Model assumes that neutrinos are massless, and because they interact so weakly with matter, they are really hard to measure directly. While the bounds it provided have been improved upon, I want to close this lecture talking about Supernova 1987a. In February of 1987, a supernova explosion was directly observed on Earth. It was observed optically and studied extensively by astronomers for several months, and was even visible to the naked eye. In the collapse of a star under its own gravity unimaginable densities are created that fuel the largest explosions in the universe. In the explosion of a supernova, the light produced in the explosion must pass through the plasma of high temperature matter. This plasma consists of ionized protons and electrons (i.e., charged particles), so the photons have a very small mean free path. They bounce around in this plasma many, many times interacting with the protons and electrons that impede them from propagating out. Photons get stuck in the supernova for an extended period of time.

Because of this, the observation of SN 1987a was really crazy. As the explosion of a star/supernova is governed by nuclear processes, we should expect copious neutrino production. Neutrinos, unlike photons, really do not like to interact with other particles.

So, while the photons are stuck in the supernova, neutrinos pass right through, with essentially no impedence. Because of the delay in the photons escaping the supernova, neutrinos from SN 1987a actually arrived at Earth before photons! There were a good 2 hours between them!

Several neutrino detection sites around the globe measured a total of 25 neutrinos from SN 1987a. This was enough, however, to place an upper bound on the mass of the neutrinos of about 16 eV. If the neutrinos were more massive, then they would have taken longer to get to Earth, and might not have beaten the light in getting here. Also, though only a whopping 25 neutrinos were observed on Earth, it is believed that 99% of the energy in a supernova explosion is ~~a~~ contained in neutrinos. In SN 1987a, these neutrinos observed on Earth were consistent with models that predicted 10^{58} neutrinos produced for a total energy of 10^{46} Joules. The sun would have to shine for about a trillion years to match the energy in neutrinos in SN 1987a!